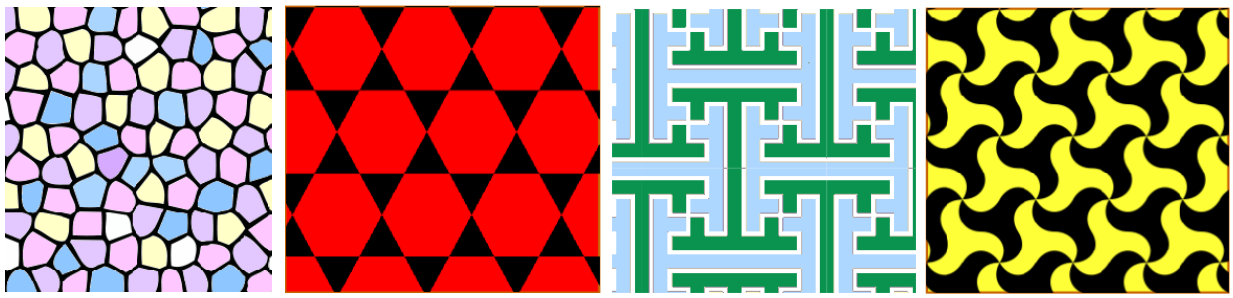
University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 6 Math Circles
Fall 2010
Tessellations I

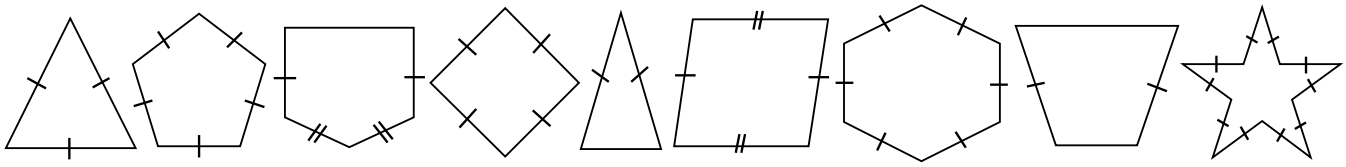
A **tessellation** is a collection of shapes that fit together with no gaps or overlaps.



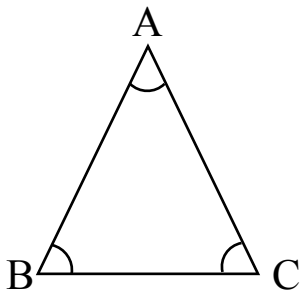
Simple Tessellations

An **interior angle** is an angle inside of a shape.
All of the sides lengths and interior angles in a **regular polygon** are equal.

Exercise 1. Which of the following shapes are regular polygons?



Interior angles of regular polygons



$\angle ABC + \angle BAC + \angle ACB = 180°$

The interior angles in *any* triangle add up to 180°.

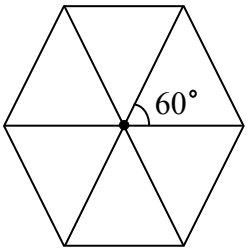
Exercise 2. Complete the following chart and determine the measure of the interior angles in each of the following regular polygons.

shape	# of triangles	angles add up to	# of vertices	interior angle
	1	$180°$	3	$\frac{180°}{3} = 60°$
	2	$180° \times 2 = 360°$	4	$\frac{360°}{4} = 90°$
				
				
				
				

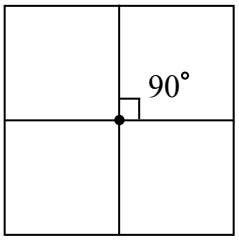
Formulas:
# of triangles = # of vertices -2
Interior angle of a regular polygon = $\frac{180° \times (\#of vertices - 2)}{\#of vertices}$

A **regular tessellation** is made up of the same shape.

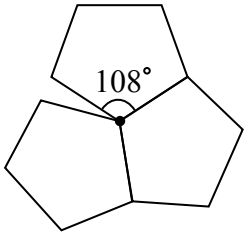
Which regular polygons can form regular tessellations?



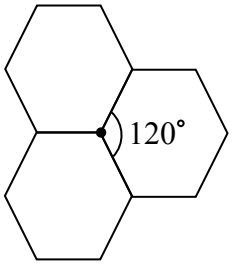
$60° \times 6 = 360°$



$90° \times 4 = 360°$



$108° \times 3 = 324°$



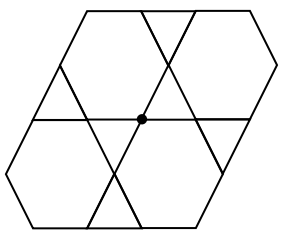
$120° \times 3 = 360°$

A regular polygon can form a regular tessellation if the measure of one of its interior angles **divides** $360°$ **evenly.**

Exercise 3. Can a regular polygon with the following number of sides form a tessellation?

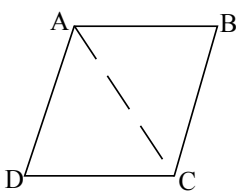
# of sides	Interior angle		Yes/No
7 sides	128.57°	360°/128.57° =	
8 sides			
9 sides			
10 sides			
11 sides			
12 sides			

Combining shapes



2 equilateral triangles + 2 regular hexagons
$= (2 \times 60°) + (2 \times 120°)$
$= 360°$

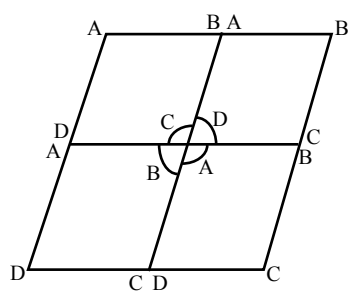
Irregular polygons



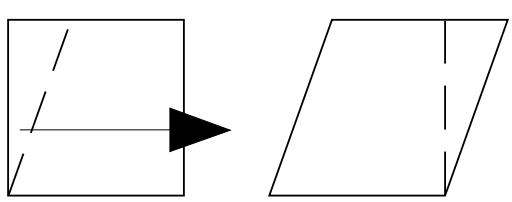
Just like a square, we can divide a parallelogram into 2 triangles. Thus, the interior angles add up to 360° (ie. $A + B + C + D = 360°$).

Can we create a regular tessellation?

Yes, since $A + B + C + D = 360°$!

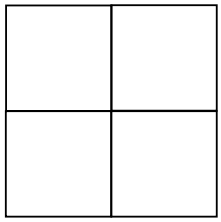


Can you see how you can create a parallelogram from a square?

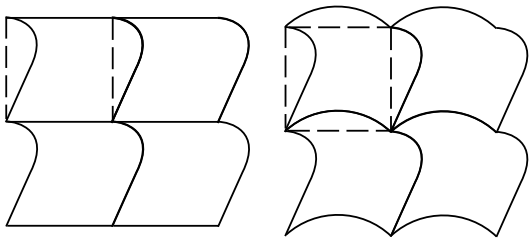


This is one way that we can create more interesting tessellations:

1. Start with a regular tessellation of a regular polygon.

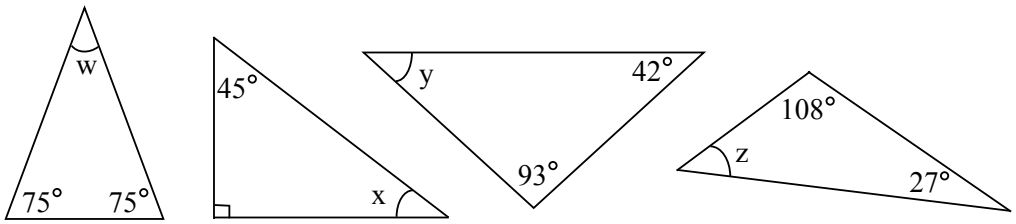


2. Cut a piece off from one side and move it to the opposite side.

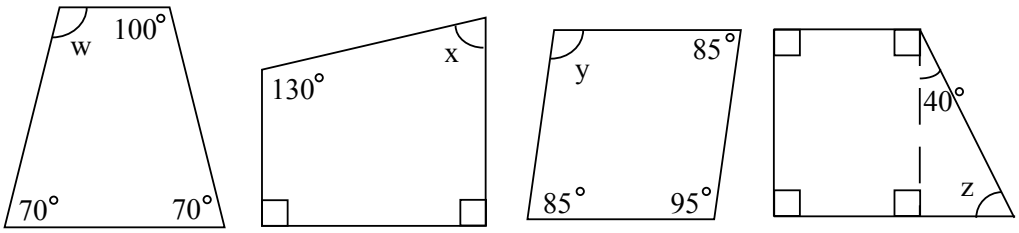


Exercises

1. Find the missing angle.

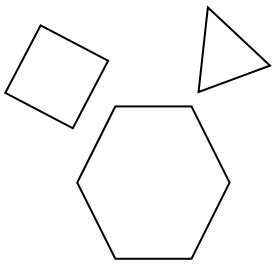


2. Find the missing angle.



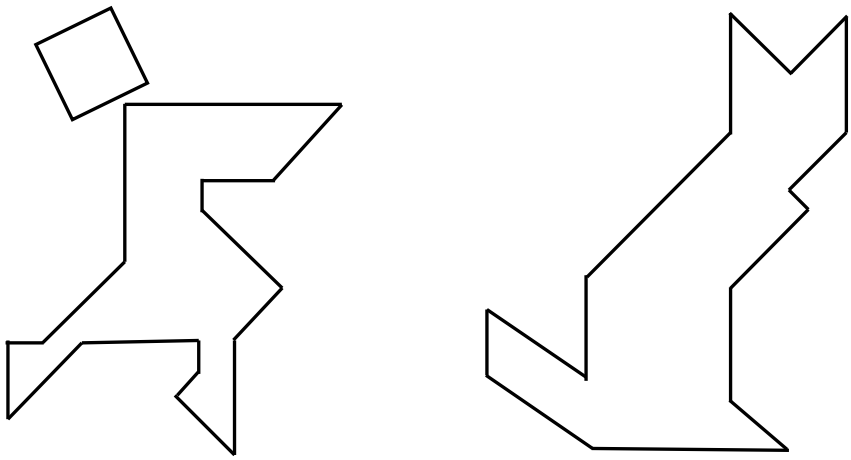
3. If you had a regular polygon with 50 sides (called a pentacontagon), what would the sum of all of the interior angles be? What is the measure of each angle?

4. Can you create a tessellation with the following regular polygons? All of the side lengths are equal and you can use as many of each shape as you need.

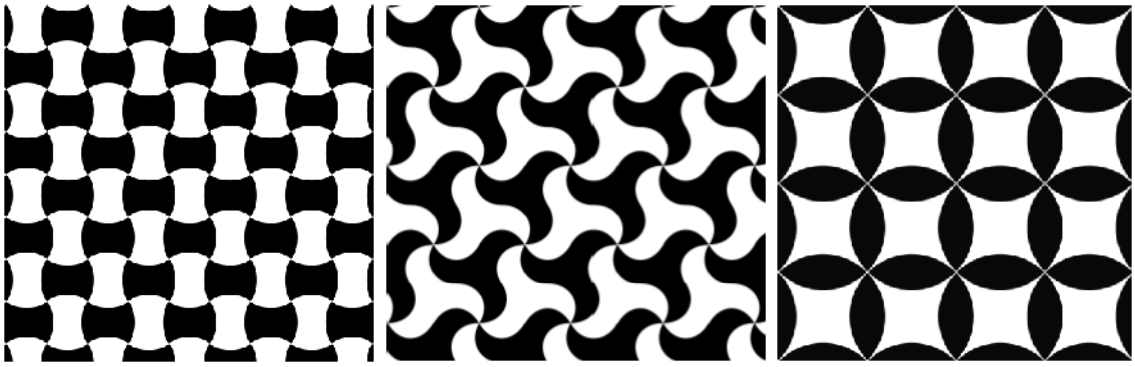


5. Is it possible to create a regular tessellation with a circle? Why or why not?

6. Divide the following shapes into smaller polygons using straight lines. Classify each of your smaller polygons as a regular or irregular polygon.

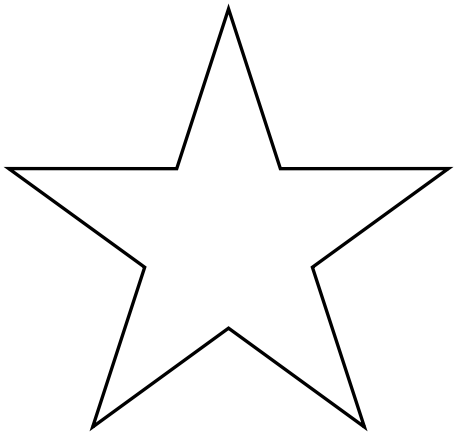


7. Can you find the base shape used to construct each of the following tessellations? (Hint: Try connecting the points where the vertices meet.)



8. Create your own regular tessellation using triangles as your base shapes.

9. What do the interior angles in a star add up to? (Hint: Try dividing it into familiar shapes.)



10. A vertex is the corner point of any shape where two lines meet. What happens when two lines meet at 180°? Does this create a vertex? Can a regular polygon have an interior angle of 180°? (Remember, all interior angles of regular polygons must be equal.)

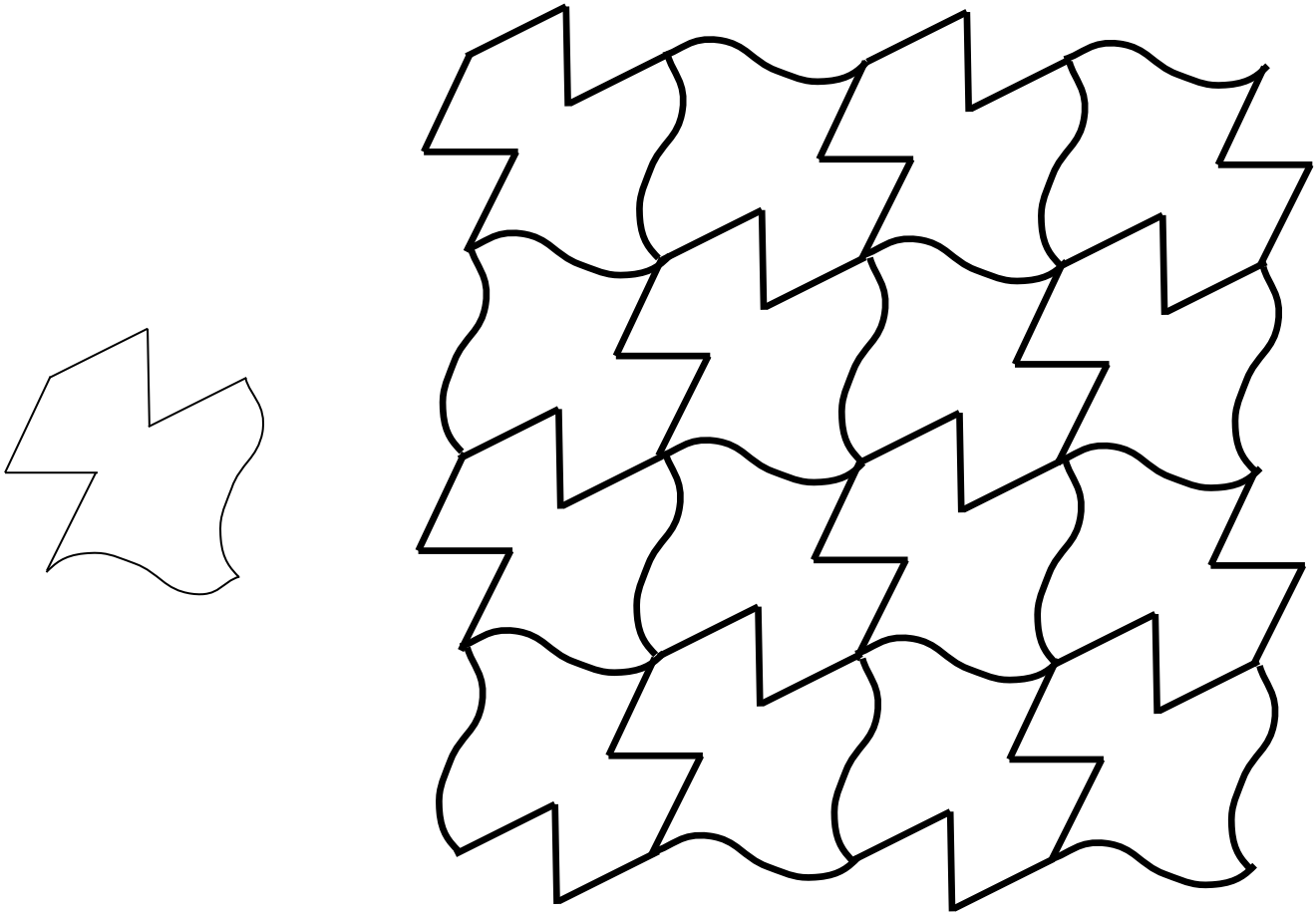
11. Claire is trying to draw a regular polygon with an interior angle less than 60°? Is this possible? (If you don't know the answer, try drawing it out.)

12. Can a regular polygon have an interior angle greater than 180°?

13. We found that the interior angles of regular polygons must divide into 360° evenly in order to create regular tessellations. What is the minimum number of shapes you need to create a regular tessellation using regular polygons? (Hint: Think about your answer to the previous question.)

14. In our investigation, we found that regular polygons with 7-12 sides could not form regular tessellations. Are there any regular polygons with greater than 12 sides that can form regular tessellations?

15. What does this shape look like? Try adding a design to the tessellation.



Solutions:

1. $180° - 75° - 75° = 30°$;
   $180° - 90° - 45° = 45°$;
   $180° - 93° - 42° = 45°$;
   $180° - 108° - 27° = 45°$

2. $360° - 100° - 70° - 70° = 120°$;
   $360° - 90° - 90° - 130° = 50°$;
   $360° - 85° - 85° - 95° = 95°$;
   $360° - 90° - 90° - 130° = 50°$

3. $180°(50 - 2) = 8640°$;
   $8640°/50 = 172.8°$

5. No; you can't fit circles together without gaps or overlaps.

7. square; triangle/hexagon/square

9. 5 Triangles + 1 pentagone $\Rightarrow$ Interior angles $= 5 \times 180° + 180°(5 - 2) = 1440°$

10. It does not create a vertex. An equilateral triangle is the only polygon that has an interior angle of $180°$

11. No, the smallest regular polygon is an equilateral triangle in which all interior angles are $60°$.

12. No, because that would mean all interior angles are greater than $180°$ which is impossible.

13. 3

14. No