University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

# Senior Math Circles
# March 11, 2009
# Geometry III

**Soap Bubbles**

Minimal Surface: a surface whose area is the smallest possible.
A bubble is a minimal surface area, and because of surface tension, soap bubbles or clusters of them, naturally try to minimize the area for the volumes they enclose.

Bubbles have fascinated people ever since the invention of soap. But the mathematics of bubbles and foam only really got going in the 1830s, when Belgian physicist Joseph A. Plateau began dipping wire frames into soap solution and was astounded by the results. Despite 170 years of research, we still have not arrived at complete mathematical explanations - or even descriptions of - several interesting phenomena that Plateau had observed. We will look at a few.

The Soap Bubble Problem is to enclose and separate $m$ regions of prescribed volumes using a singular surface of minimal area.

Double Bubble Theorem:
The least-area enclosure of two prescribed volumes in $R^3$ is the double bubble.

Double Bubble Problem:
The shape formed when two bubbles coalesce consists of three spherical surfaces. These three pieces meet at angles of 120°. There are two cases of shapes for this problem:

If the two volumes are equal, the middle sphere is a flat disc, as shown. This was recently proved in 1995 by Joel Hass.

The problem of unequal volumes remains an open problem.

Triple Bubble Problem:
What happens when three bubbles coalesce?
The triple bubble problem in $R^3$ currently remains an unsolved problem, on which hundreds of brilliant mathematicians currently research.

Exercise:
Make a conjecture about what you think would happen in the Triple Bubble Problem.

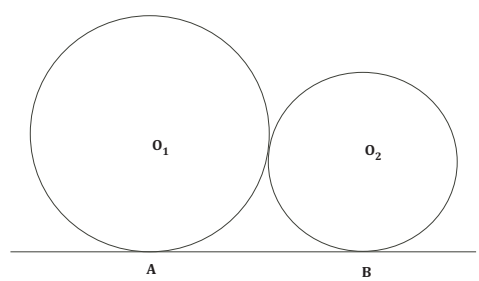
Exercise:
You can easily create your own bubbles and get a first hand experience on how bubbles form and come together to create surfaces. All you need is some wire which you can bend into different shapes. By dipping this wire in a soap solution you can then create your own mathematical bubbles.

**Japanese Temple Geometry**

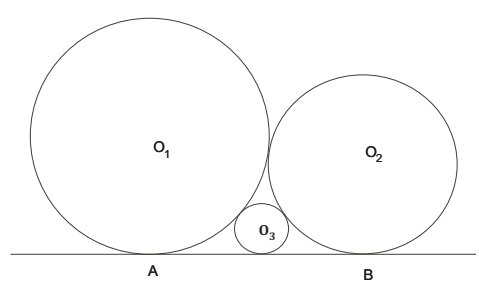
During the Edo period in Japan from $1603 - 1867$ Japan was almost completely isolated from the Western World. This meant that no books on mathematics arrived in Japan at this time. Despite this, during this period of isolation, people of all social classes produced theorems in Euclidean Geometry.
These theorems were not published in books, but appeared as beautifully colored drawings on wooden tablets which were hung under the roofs in the precincts of a shrine or temple. Tablets would sometimes contain five or six theorems, and the name and social rank of the presenter was always given. Proofs of theorems were rarely given, and there was no descriptions of the figure, just references to it.
The geometers in this era were superbly skilled, especially in the area of algebraic manipulation. Many solutions did not even use trigonometry and were remarkably involved. Some theorems would be very difficult to prove by Western methods.
Many of the tablets were lost, but over 900 remain.

**Japanese Temple Problems**

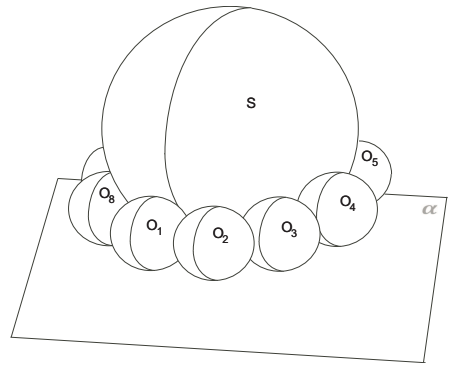
1. The circles $O_1$ and $O_2$, radius $r_1, r_2$, touch each other externally and the line $l$ at the points $A$ and $B$ respectively. Show that $(AB)^2 = 4r_1r_2$.



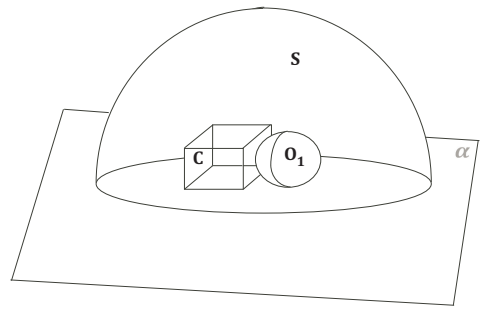

2. A circle $O_3$ of radius $r_3$ touches the line $l$ and the circles described in the above example. Show that $\frac{1}{\sqrt{r_3}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}$



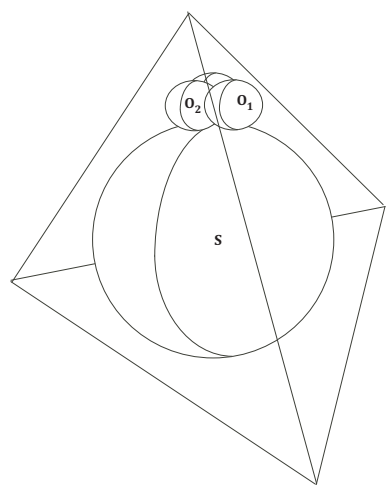
3. A sphere $S$ of radius $r$ touches a plane $\alpha$, and the spheres $O_i (i = 1, 2..., 8)$ of radius $t$ form a loop of contact spheres of equal size which all touch $\alpha$ and $S$. Find the relation between $t$ and $r$.




4. The base of hemisphere $S$, with radius $r$, lies in a plane $\alpha$, and a cube $C$ of side $t$ has one face in $\alpha$, and 4 vertices lie on the hemisphere.
Find $t$ in terms of $r$, and find the radius $r_1$ of the sphere which touches $\alpha$, a face of the cube at the centre, and also touches the hemisphere internally, in terms of $r$.




5. $S$ is a sphere of radius $r$ inscribed in a regular tetrahedron. $O_i (i = 1, 2, 3)$ form a loop of equal spheres, of radius $t$, which touch $S$ externally, each also touching one face of the tetrahedron, as well as each other. Find $t$ in terms of $r$.



Works Cites:

"Japanese Temple Problems: San Gaku" H Fukagawa and D. Pedoe.