The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fryer Contest

(Grade 9)

Thursday, April 18, 2013
(in North America and South America)

Friday, April 19, 2013
(outside of North America and South America)

UNIVERSITY OF **WATERLOO**   **WATERLOO** MATHEMATICS   **Deloitte.**

Do not open this booklet until instructed to do so.

Time: 75 minutes   **Number of questions: 4**
Calculators are permitted   **Each question is worth 10 marks**

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 💡

   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by ✎

   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

   - Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
   - Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\ldots$ or $1.41\ldots$, except where otherwise indicated.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our Web site, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

1. Ann, Bill and Cathy went bowling. In bowling, each score is a whole number.

    (a) In Ann's first game, her score was 103. In her second game, her score was 117. What was her average score for these two games?

    (b) In his first two games, Bill's scores were 108 and 125. His average score after three games was 115. What was his score in the third game?

    (c) After three games, Cathy's average score was 113. She scored the same in her fifth game as she did in her fourth game. Is it possible for her average score on these five games to be 120? Explain why or why not.

2. The outside of a field consists of two straight sides each of length 100 m joined by two semi-circular arcs each of diameter 60 m, as shown below.
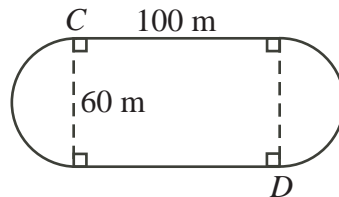

    (a) Determine the perimeter of the field.

    (b) Amy and Billais run from point C to point D. Amy runs along the perimeter of the field, and Billais runs in a straight line from C to D. Rounded to the nearest metre, how much farther does Amy travel than Billais?

    (c) The diagram below shows a track of constant width x m built around the field. The outside of the track has two straight sides each of length 100 m joined by two semi-circular arcs. The perimeter of the outside of the track is 450 m. Determine the value of x rounded to the nearest whole number.
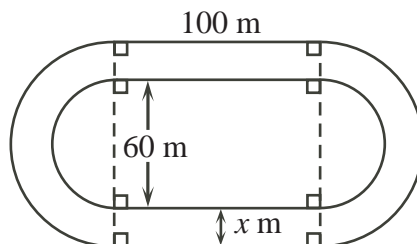
3. The *sum of the digits* of 2013 is $2 + 0 + 1 + 3 = 6$. If the sum of the digits of a positive integer is divisible by 3, then the number is divisible by 3. Also, if a positive integer is divisible by 3, then the sum of its digits is divisible by 3.

   (a) List all values for the digit A such that the four-digit number $51A3$ is divisible by 3.

   (b) List all values for the digit B such that the four-digit number $742B$ is divisible by both 2 and 3 (that is, is divisible by 6).

   (c) Find all possible pairs of digits P and Q such that the number $1234PQPQ$ is divisible by 15.

   (d) Determine the number of pairs of digits C and D for which the product $2CC \times 3D5$ is divisible by 12.

4. A dot starts on the xy-plane at $(0,0)$ and makes a series of moves.
   In each move, the dot travels one unit either left ($\leftarrow$), right ($\rightarrow$), up ($\uparrow$), or down ($\downarrow$).

   Five of the many different ways that the dot could end at the point $(1,1)$ are $\uparrow\rightarrow$, $\rightarrow\uparrow$, $\uparrow\downarrow\rightarrow\uparrow$, $\uparrow\uparrow\rightarrow\downarrow$, and $\uparrow\rightarrow\rightarrow\leftarrow$.

   (a) In how many different ways can the dot end at the point $(1,0)$ in 4 or fewer moves?

   (b) At how many different points can the dot end in exactly 4 moves?

   (c) Determine, with justification, the number of integers k with $k \leq 100$ for which the dot can end at the point $(-7, 12)$ in exactly k moves.

   (d) The dot can end at exactly 2304 points in exactly 47 moves. Determine, with justification, the number of points at which the dot can end in exactly 49 moves.

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Mathematics Contest or the Canadian Senior Mathematics Contest,
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Fryer Contest

(Grade 9)

Thursday, April 12, 2012
(in North America and South America)

Friday, April 13, 2012
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

WATERLOO
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TIPS: 1.   Please read the instructions on the front cover of this booklet.
        2.   Write all answers in the answer booklet provided.

        3.   For questions marked 💡, place your answer in the appropriate box in the answer booklet and **show your work**.

        4.   For questions marked 📐, provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
        5.   Diagrams are *not* drawn to scale. They are intended as aids only.

1. 💡 (a) In Carrotford, candidate A ran for mayor and received 1008 votes out of a total of 5600 votes. What percentage of all votes did candidate A receive?

    💡 (b) In Beetland, exactly three candidates, B, C and D, ran for mayor. Candidate B won the election by receiving $\frac{3}{5}$ of all votes, while candidates C and D tied with the same number of votes. What percentage of all votes did candidate C receive?

    📐 (c) In Cabbagetown, exactly two candidates, E and F, ran for mayor and 6000 votes were cast. At 10:00 p.m., only 90% of these votes had been counted. Candidate E received 53% of those votes. How many more votes had been counted for candidate E than for candidate F at 10:00 p.m.?

    📐 (d) In Peaville, exactly three candidates, G, H and J, ran for mayor. When all of the votes were counted, G had received 2000 votes, H had received 40% of the votes, and J had received 35% of the votes. How many votes did candidate H receive?

2. The *prime factorization* of 144 is $2 \times 2 \times 2 \times 2 \times 3 \times 3$ or $2^4 \times 3^2$. Therefore, 144 is a perfect square because it can be written in the form $(2^2 \times 3) \times (2^2 \times 3)$.
    The prime factorization of 45 is $3^2 \times 5$. Therefore, 45 is not a perfect square, but 45×5 is a perfect square, because $45 \times 5 = 3^2 \times 5^2 = (3 \times 5) \times (3 \times 5)$.

    💡 (a) Determine the prime factorization of 112.

    💡 (b) The product $112 \times u$ is a perfect square. If u is a positive integer, what is the smallest possible value of u?

    📐 (c) The product $5632 \times v$ is a perfect square. If v is a positive integer, what is the smallest possible value of v?

    📐 (d) A *perfect cube* is an integer that can be written in the form n^3, where n is an integer. For example, 8 is a perfect cube since $8 = 2^3$. The product $112 \times w$ is a perfect cube. If w is a positive integer, what is the smallest possible value of w?

3. The positive integers are arranged in rows and columns, as shown, and described below.

	A	B	C	D	E	F	G
Row 1		1	2	3	4	5	6
Row 2	12	11	10	9	8	7	
Row 3		13	14	15	16	17	18
Row 4	24	23	22	21	20	19	

$$\vdots$$

The odd numbered rows list six positive integers in order from left to right beginning in column B. The even numbered rows list six positive integers in order from right to left beginning in column F.

(a) Determine the largest integer in row 30.

(b) Determine the sum of the six integers in row 2012.

(c) Determine the row and column in which the integer 5000 appears.

(d) For how many rows is the sum of the six integers in the row greater than 10 000 and less than 20 000?

4. The volume of a cylinder with radius r and height h equals $\pi r^2 h$.
The volume of a sphere with radius r equals $\frac{4}{3}\pi r^3$.

(a) The diagram shows a sphere that fits exactly inside a cylinder. That is, the top and bottom faces of the cylinder touch the sphere, and the cylinder and the sphere have the same radius, r. State an equation relating the height of the cylinder, h, to the radius of the sphere, r.
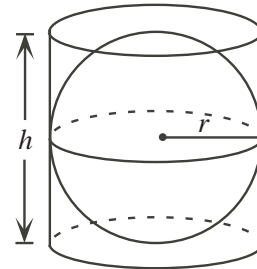
(b) For the cylinder and sphere given in part (a), determine the volume of the cylinder if the volume of the sphere is 288π.

(c) A solid cube with edges of length 1 km is fixed in outer space. Darla, the baby space ant, travels on this cube and in the space around (but not inside) this cube. If Darla is allowed to travel no farther than 1 km from the nearest point on the cube, then determine the total volume of space that Darla can occupy.

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1. An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant d, called the common difference. For example, $2, 5, 8, 11, 14$ are the first five terms of an arithmetic sequence with a common difference of $d = 3$.

    (a) Determine the 6th and 7th terms of the sequence given above.

    (b) What is the 31st term in this sequence?

    (c) If the last term in this sequence were 110, how many terms would there be in the sequence?

    (d) If this sequence is continued, does 1321 appear in the sequence? Explain why or why not.

2. In any isosceles triangle ABC with $AB = AC$, the altitude AD bisects the base BC so that $BD = DC$.
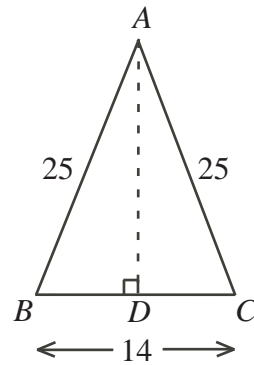
    (a) (i) As shown in $\triangle ABC$, $AB = AC = 25$ and $BC = 14$. Determine the length of the altitude AD.

    (ii) Determine the area of $\triangle ABC$.

    (b) Triangle ABC from part (a) is cut along its altitude from A to D (Figure 1).
    Each of the two new triangles is then rotated $90°$ about point D until B meets C directly below D (Figure 2).
    This process creates the new triangle which is labelled PQR (Figure 3).
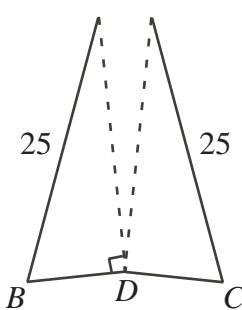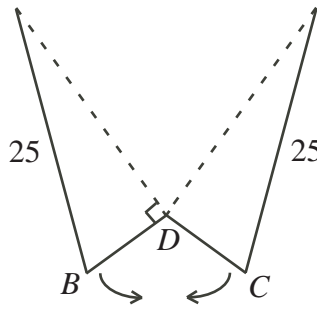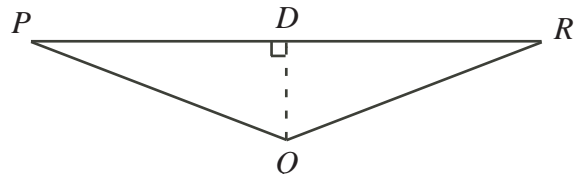

Figure 1   Figure 2   Figure 3

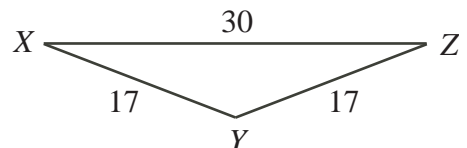   (i) In $\triangle PQR$, determine the length of the base PR.
    (ii) Determine the area of $\triangle PQR$.

    (c) There are two different isosceles triangles whose side lengths are integers and whose areas are 120. One of these two triangles, $\triangle XYZ$, is shown. Determine the lengths of the three sides of the second triangle.

3. Begin with any two-digit positive integer and multiply the two digits together. If the resulting product is a two-digit number, then repeat the process. When this process is repeated, all two-digit numbers will eventually become a single digit number. Once a product results in a single digit, the process stops.

For example,

| Two-digit number	Step 1	Step 2	Step 3	
97	$9 \times 7 = 63$	$6 \times 3 = 18$	$1 \times 8 = 8$	The process stops at 8 after 3 steps.
48	$4 \times 8 = 32$	$3 \times 2 = 6$		The process stops at 6 after 2 steps.
50	$5 \times 0 = 0$			The process stops at 0 after 1 step.

   (a) Beginning with the number 68, determine the number of steps required for the process to stop.

   (b) Determine all two-digit numbers for which the process stops at 8 after 2 steps.

   (c) Determine all two-digit numbers for which the process stops at 4.

   (d) Determine a two-digit number for which the process stops after 4 steps.

4. Ian buys a cup of tea every day at Jim Bortons for $1.72 with money from his coin jar. He starts the year with 365 two-dollar (200¢) coins and no other coins in the jar. Ian makes payment and the cashier provides change according to the following rules:

   • Payment is only with money from the coin jar.

   • The amount Ian offers the cashier is at least $1.72.

   • The amount Ian offers the cashier is as close as possible to the price of the cup of tea.

   • Change is given with the fewest number of coins.

   • Change is placed into the coin jar.

   • Possible coins that may be used have values of 1¢, 5¢, 10¢, 25¢, and 200¢.

   (a) How much money will Ian have in the coin jar after 365 days?

   (b) What is the maximum number of 25¢ coins that Ian could have in the coin jar at any one time?

   (c) How many of each type of coin does Ian have in his coin jar after 277 days?

2010 Fryer Contest (Grade 9)
Friday, April 9, 2010

1. Consider the following sequence of figures showing arrangements of square tiles:

Figure 1   Figure 2   Figure 3   Figure 4
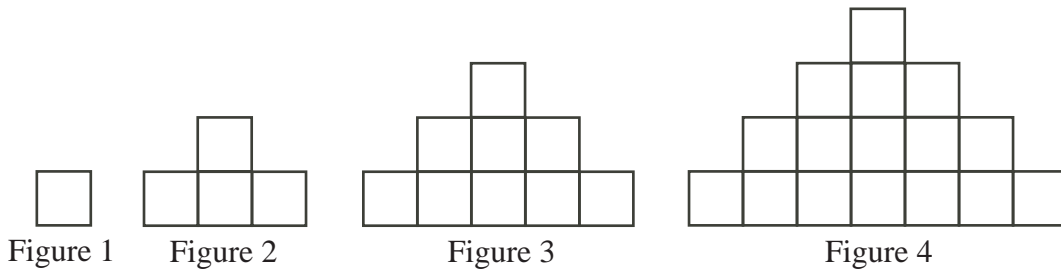
More figures can be drawn, each having one row of tiles more than the previous figure. This new bottom row is constructed using two tiles more than the number of tiles in the bottom row of the previous figure.

(a) Figure 4 is cut into two pieces as shown. Draw a rearrangement of these two pieces showing how they can be formed into a square having $4^2 = 16$ tiles.
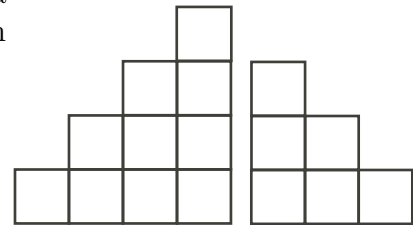
(b) Determine the number of tiles in Figure 5.

(c) Determine the number of tiles in the bottom row of Figure 10.

(d) Determine the difference between the total number of tiles in Figure 11 and the total number of tiles in Figure 9.

2. (a) Determine the average of the integers $71, 72, 73, 74, 75$.

   (b) Suppose that $n, n + 1, n + 2, n + 3, n + 4$ are five consecutive integers.

      (i) Determine a simplified expression for the sum of these five consecutive integers.
      (ii) If the average of these five consecutive integers is an odd integer, explain why n must be an odd integer.

   (c) Six consecutive integers can be represented by $n, n + 1, n + 2, n + 3, n + 4, n + 5$, where n is an integer. Explain why the average of six consecutive integers is never an integer.

3. Train 1 is travelling from Amville to Batton
   at a constant speed.
   Train 2 is travelling from Batton to Amville
   at a constant speed.

   (a) Train 1 travels at 60 km/h and travels $\frac{2}{3}$ of the distance to Batton in 9 hours.
       Determine the distance from Amville to Batton.

   (b) Train 2 travels $\frac{2}{3}$ of the distance to Amville in 6 hours. How fast is the train going?

   (c) Train 2 started its trip $3\frac{1}{2}$ hours after Train 1 started its trip. Both trains arrived at
       Cuford at 9:00 p.m. What time did Train 1 leave Amville?

4. A *palindrome* is a positive integer that is the same when read forwards or backwards. For
   example, three palindromes are $7, 121$ and 7739377.

   (a) Determine the number of palindromes less than 1000.

   (b) Determine the number of palindromes with 7 digits.

   (c) If the palindromes in part (b) are written in increasing order, determine the 2125th
       palindrome in the list.

   (d) Determine the number of six-digit palindromes that are divisible by 91.

# *2009 Fryer Contest (Grade 9)*
Wednesday, April 8, 2009

1. Emily sets up a lemonade stand. She has set-up costs of $12.00 and each cup of lemonade costs her $0.15 to make. She sells each cup of lemonade for $0.75.

    (a) What is the total cost, including the set-up, for her to make 100 cups of lemonade?

    (b) What is her profit (money earned minus total cost) if she sells 100 cups of lemonade?

    (c) What is the number of cups that she must sell to break even (that is, to have a profit of $0)?

    (d) Why is it not possible for her to make a profit of exactly $17.00?

2. If $a > 0$ and $b > 0$, a new operation ∇ is defined as follows: $a \nabla b = \dfrac{a+b}{1+ab}$.

    For example, $3 \nabla 6 = \dfrac{3+6}{1+3 \times 6} = \dfrac{9}{19}$.

    (a) Calculate $2 \nabla 5$.

    (b) Calculate $(1 \nabla 2) \nabla 3$.

    (c) If $2 \nabla x = \dfrac{5}{7}$, what is the value of x?

    (d) For some values of x and y, the value of $x \nabla y$ is equal to $\dfrac{x+y}{17}$. Determine all possible ordered pairs of positive integers x and y for which this is true.

3. In the diagram, K, O and M are the centres of the three semi-circles. Also, $OC = 32$ and $CB = 36$.

    (a) What is the length of AC?

    (b) What is the area of the semi-circle with centre K?

    (c) What is the area of the shaded region?

    (d) Line l is drawn to touch the smaller semi-circles at points S and E so that KS and ME are both perpendicular to l. Determine the area of quadrilateral $KSEM$.
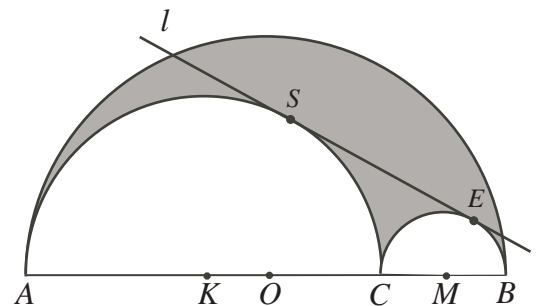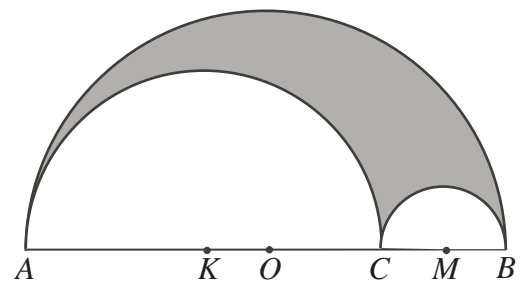
4. The addition shown below, representing $2 + 22 + 222 + 2222 + \cdots$, has 101 rows and the last term consists of 101 2's:

$$
\begin{array}{cccccccccc}
 & & & & & & & & & 2 \\
 & & & & & & & & 2 & 2 \\
 & & & & & & & 2 & 2 & 2 \\
 & & & & & & 2 & 2 & 2 & 2 \\
 & & & & & & & & & \vdots \\
 & & 2 & 2 & \cdots & 2 & 2 & 2 & 2 \\
+ & 2 & 2 & 2 & \cdots & 2 & 2 & 2 & 2 \\
\hline
 & & & & & \cdots & C & B & A
\end{array}
$$

(a) Determine the value of the ones digit A .

(b) Determine the value of the tens digit B and the value of the hundreds digit C.

(c) Determine the middle digit of the sum.

2008 Fryer Contest (Grade 9)
Wednesday, April 16, 2008

1. A *magic square* is a grid of numbers in which the sum of the numbers in each row, in each column, and on each of the two main diagonals is equal to the same number (called the *magic constant*). For example,

4	3	8
9	5	1
2	7	6

is a magic square because the sum of the numbers in each row, in each column, and on each of the two main diagonals is equal to 15. (15 is the magic constant.)

   (a) A magic square is to be formed using the nine integers from 11 to 19.

      (i) Calculate the sum of the nine integers from 11 to 19.
      (ii) Determine the magic constant for this magic square and explain how you found it.
      (iii) Complete the magic square starting with the entries

18	11	
	12	

   (b) A magic square is to be formed using the sixteen integers from 1 to 16.

      (i) Calculate the sum of the sixteen integers from 1 to 16.
      (ii) Determine the magic constant for this magic square and explain how you found it.
      (iii) Complete the magic square starting with the entries

16	3		13
5		11	
		7	12
4		14	1

2. If a team won 13 games and lost 7 games, its *winning percentage* was $\dfrac{13}{13+7} \times 100\% = 65\%$, because it won 13 of its 20 games.

   (a) The Sharks played 10 games and won 8 of these.
   Then they played 5 more games and won 1 of these.
   What was their final winning percentage? Show the steps that you took to find your answer.

   (b) The Emus won 4 of their first 10 games.
   The team played x more games and won all of these.
   Their final winning percentage was 70%.
   How many games did they play in total? Show the steps that you took to find your answer.

   (c) The Pink Devils started out the season with 7 wins and 3 losses.
   They lost all of their games for the rest of the season.
   Was there a point during the season when they had won exactly $\frac{2}{7}$ of their games? Explain why or why not.

3. (a) Figure 1 shows a net that can be folded to create a rectangular box. Determine the volume and the surface area of the box.
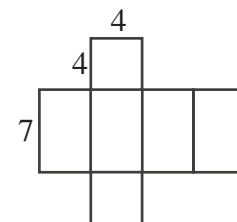

Figure 1

(b) In Figure 2, the rectangular box has dimensions 2 by 2 by 6. From point A, an ant walked to point B crossing all four of the side faces. The shortest path along which the ant could walk may be found by unfolding the box, as in Figure 3, and drawing a straight line from A to B. Determine the length of AB in Figure 3.
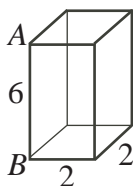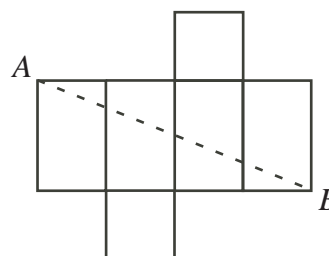

Figure 2

Figure 3

(c) In Figure 4, the rectangular block has dimensions 3 by 4 by 5. A caterpillar is at corner A. Determine, with justification, the shortest possible distance from A to G along the surface of the block.
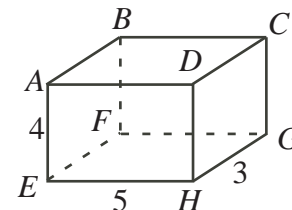

Figure 4

4. When the first 30 positive integers are written together in order, the 51-digit number

$$x = 123456789101112131415161718192021222324252627282930$$

is formed.

(a) A positive integer that is the same when read forwards or backwards is called a *palindrome*. For example, 12321 and 1221 are both palindromes.
Determine the smallest number of digits that must be removed from x so that the remaining digits can be rearranged to form a palindrome. Justify why this is the minimum number of digits.

(b) Determine the minimum number of digits that must be removed from x so that the remaining digits have a sum of 130. Justify why this is the minimum number of digits.

(c) When the first 50 positive integers are written in order, the 91-digit number

$$y = 123456789101112 \cdots 484950$$

is formed. Determine the minimum number of digits that must be removed from y so that the remaining digits have a sum of 210 and can be rearranged to form a palindrome. Justify why this is the minimum number of digits.

2007 Fryer Contest (Grade 9)
Wednesday, April 18, 2007
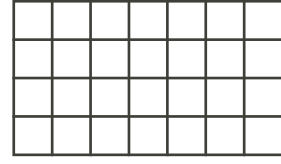
1. Squares measuring 1 by 1 are arranged to form the following sequence of rectangles:
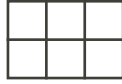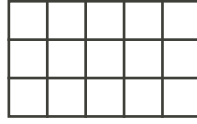

Rectangle 1   Rectangle 2   Rectangle 3

Many more rectangles are drawn, with each rectangle having one more row and two more columns than the previous rectangle.

(a) How many 1 by 1 squares would there be in Rectangle 4? Explain how you obtained your answer.

(b) Determine the perimeter of Rectangle 4. Explain how you obtained your answer.

(c) Determine the perimeter of Rectangle 7. Explain how you obtained your answer.

(d) Rectangle n has a perimeter of 178. Determine n. Explain how you obtained your answer.

2. At last week's hockey game involving the Waterloo Blueberries, the price of a platinum ticket was $25, the price of a gold ticket was $10, the price of a silver ticket was $5, and the price of a bronze ticket was $1.

(a) Jim buys 5 platinum tickets, 2 gold tickets and 3 silver tickets. Determine the average cost of the tickets that Jim buys. Explain how you obtained your answer.

(b) Mike buys 8 tickets whose average cost is $12. He then buys five more platinum tickets. What is the new average cost of the tickets that he has bought? Explain how you obtained your answer.

(c) Ophelia buys 10 tickets with an average cost of $14. Suppose that she buys n more platinum tickets. The new average cost of the tickets that she has bought is $20. What is the value of n? Explain how you obtained your answer.

3. (a) A number is divisible by 8 if the number formed by its last 3 digits is divisible by 8. For example, the number $47\,389\,248$ is divisible by 8 because 248 is divisible by 8. However, $47\,389\,284$ is not divisible by 8 because 284 is not divisible by 8.

If $992\,466\,1A6$ is divisible by 8, where A represents one digit, what are the possible values of A? Explain how you obtained your answer.

(b) A number is divisible by 9 if the sum of its digits is divisible by 9. For example, the number $19\,836$ is divisible by 9 but $19\,825$ is not.

If $D\,767\,E89$ is divisible by 9, where D and E each represent a single digit, what are the possible values of the sum $D + E$? Explain how you obtained your answer.

(c) The number $5\,41G\,507\,2H6$ is divisible by 72. If G and H each represent a single digit, what pairs of values of G and H are possible? Explain how you obtained your answer.

4. (a) In the diagram, $\triangle XYZ$ is right-angled at X, with $YX = 60$ and $XZ = 80$. W is the point on YZ so that WX is perpendicular to YZ. Determine the length of WZ. Explain how you obtained your answer.
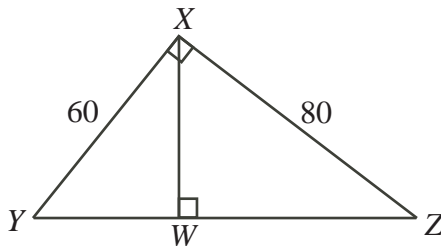
  (b) Five points A, B, C, D, and O lie on a flat field. A is directly north of O, B is directly west of O, C is directly south of O, and D is directly east of O. The distance between C and D is 140 m. A hot-air balloon is positioned in the air at H directly above O. The balloon is held in place by four ropes HA, HB, HC, and HD. Rope HC has length 150 m and rope HD has length 130 m. Determine how high the balloon is above the field (that is, determine the length of OH). Explain how you obtained your answer.
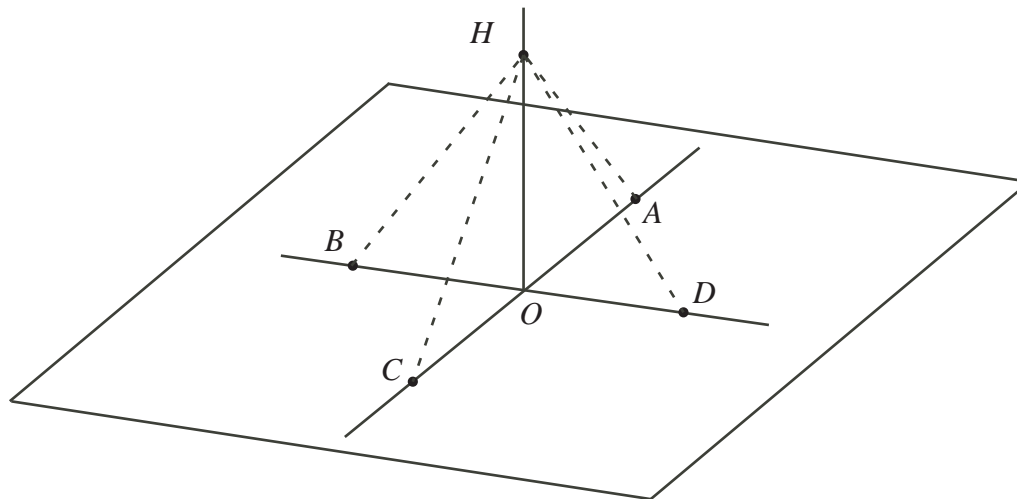

  (c) To reduce the total length of rope used, rope HC and rope HD are to be replaced by a single rope HP where P is a point on the straight line between C and D. (The balloon remains at the same position H above O as in part (b).) Determine the greatest length of rope that can be saved. Explain how you obtained your answer.

2006 Fryer Contest (Grade 9)
Thursday, April 20, 2006

1. Samantha receives the following marks out of 100 in seven of her eight courses:

Math	94
Science	93
English	84
Art	81
History	74
Phys Ed	83
Geography	79

   (a) Determine her average mark in these seven courses.

   (b) Before she finds out her actual French mark, Samantha calculates the highest possible average that she could obtain in all eight courses. Determine this average.

   (c) When Samantha actually finds out her French mark, it turns out that her average in all eight courses is 85. What is her actual French mark?

2. Dmitri has a collection of identical cubes. Each cube is labelled with the integers 1 to 6 as shown in the following net:
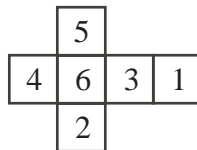

(This net can be folded to make a cube.)
He forms a pyramid by stacking layers of the cubes on a table, as shown, with the bottom layer being a 7 by 7 square of cubes.
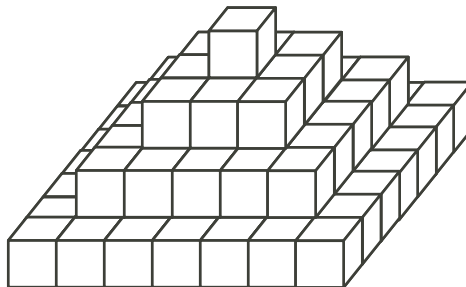

   (a) Determine the total number of cubes used to build the pyramid. Explain how you got your answer.

   (b) How many faces are visible after the pyramid is built and sitting on the table?

   (c) Explain in detail how he should position the cubes so that if all of the visible numbers are added up, the total is as large as possible. What is this total?

3. Three congruent isosceles triangles DAO, AOB and OBC have $AD = AO = OB = BC = 10$ and $AB = DO = OC = 12$. These triangles are arranged to form trapezoid $ABCD$, as shown. Point P is on side AB so that OP is perpendicular to AB.
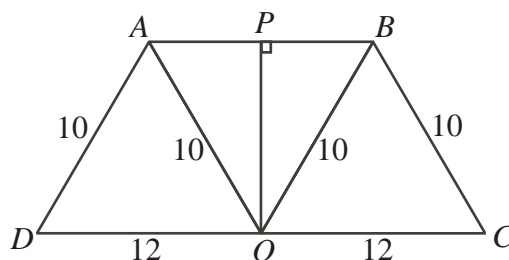

   (a) What is the length of OP? Explain how you got your answer.

   (b) What is the area of trapezoid $ABCD$? Explain how you got your answer.

   (c) Point X is the midpoint of AD and point Y is the midpoint of BC. When X and Y are joined, the trapezoid is divided into two smaller trapezoids. What is the ratio of the area of trapezoid $ABYX$ to the area of trapezoid $XYCD$? Explain how you got your answer.
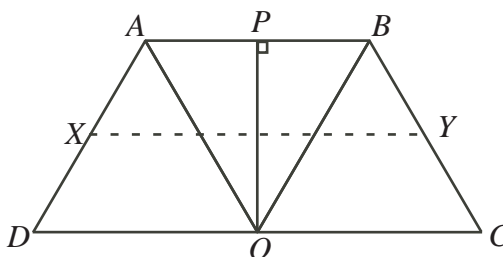

4. (a) How many of the positive integers from 1 to 100, inclusive, do not contain the digit 7? Explain how you got your answer.

   (b) How many of the positive integers from 1 to 2000, inclusive, do not contain the digit 7? Explain how you got your answer.

   (c) Determine the sum of all of the positive integers from 1 to 2006, inclusive, that do not contain the digit 7. Explain how you got your answer.

1. (a) Two circles have the same centre C. (Circles which have the same centre are called *concentric*.) The larger circle has radius 10 and the smaller circle has radius 6. Determine the area of the ring between these two circles.
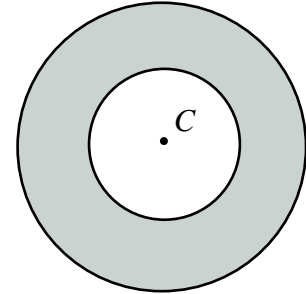
   (b) In the diagram, the three concentric circles have radii of 4, 6 and 7. Which of the three regions X, Y or Z has the largest area? Explain how you got your answer.
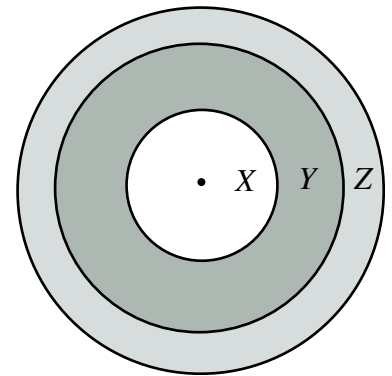
   (c) Three concentric circles are shown. The two largest circles have radii of 12 and 13. If the area of the ring between the two largest circles equals the area of the smallest circle, determine the radius of the smallest circle. Explain how you got your answer.
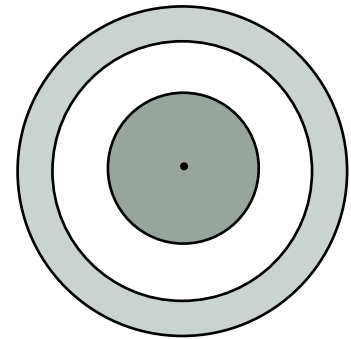
2. A game begins with a row of empty boxes. On a turn, a player can put his or her initial in 1 box or in 2 adjacent boxes. (Boxes are called *adjacent* if they are next to each other.) Anh and Bharati alternate turns. Whoever initials the last empty box wins the game.

   (a) The game begins with a row of 3 boxes. Anh initials the middle box. Explain why this move guarantees him a win no matter what Bharati does.
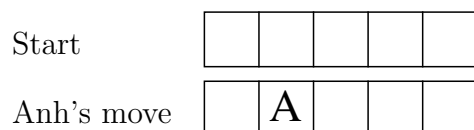
(b) Now the game begins with a row of 5 boxes. Suppose that the following moves have occurred:

Start

Anh's move
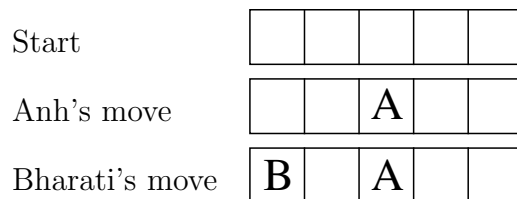
Bharati's move

Show a move that Anh can make next in order to guarantee that he will win. Explain how this move prevents Bharati from winning.

(c) Again the game begins with a row of 5 boxes. Suppose that the following move has occurred.

Start

Anh's move

Show the two possible moves that Bharati can make next to guarantee she wins. Explain how each of these moves prevents Anh from winning.

3. A *Nakamoto triangle* is a right-angled triangle with *integer* side lengths which are in the ratio $3 : 4 : 5$. (For example, a triangle with side lengths 9, 12 and 15 is a Nakamoto triangle.)

   (a) If one of the sides of a Nakamoto triangle has length 28, what are the lengths of the other two sides?

   (b) Find the lengths of the sides of the Nakamoto triangle which has perimeter 96. Explain how you got your answer.

   (c) Determine the area of each of the Nakamoto triangles which has a side of length 60. Explain how you got your answers.

4. Points B, C, and D lie on a line segment AE, as shown.

$A \qquad B \qquad\quad C \qquad\qquad\quad D \qquad\qquad E$
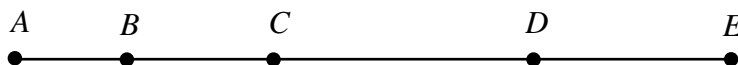
The line segment AE has 4 *basic sub-segments* AB, BC, CD, and DE, and 10 *sub-segments*: AB, AC, AD, AE, BC, BD, BE, CD, CE, and DE.
The *super-sum* of AE is the sum of the lengths of all of its sub-segments.

   (a) If $AB = 3$, $BC = 6$, $CD = 9$, and $DE = 7$, determine the lengths of the 10 sub-segments, and calculate the super-sum of AE.

   (b) Explain why it is impossible for the line segment AE to have 10 sub-segments of lengths 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

   (c) When the super-sum of a new line segment AJ with 9 basic sub-segments of lengths from left to right of $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$ is calculated, the answer is 45.
   Determine the super-sum of a line segment AP with 15 basic sub-segments of lengths from left to right of $1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{15}$.

2004 Fryer Contest (Grade 9)
Thursday, April 15, 2004

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1. Lloyd is practising his arithmetic by taking the reciprocal of a number and by adding 1 to a number. Taking the reciprocal of a number is denoted by $\xrightarrow{R}$ and adding 1 is denoted by $\xrightarrow{A}$.
   Here is an example of Lloyd's work, starting with an input of 2:
   $$2 \xrightarrow{R} \frac{1}{2} \xrightarrow{A} \frac{3}{2} \xrightarrow{R} \frac{2}{3} \xrightarrow{A} \frac{5}{3} \xrightarrow{R} \frac{3}{5}$$

   (a) Using an input of 3, fill in the five blanks below:
   $$3 \xrightarrow{R} \underline{\quad} \xrightarrow{A} \underline{\quad} \xrightarrow{R} \underline{\quad} \xrightarrow{A} \underline{\quad} \xrightarrow{R} \underline{\quad}$$

   (b) Using an input of x, use the same operations and fill in the five blanks below:
   $$x \xrightarrow{R} \underline{\quad} \xrightarrow{A} \underline{\quad} \xrightarrow{R} \underline{\quad} \xrightarrow{A} \underline{\quad} \xrightarrow{R} \underline{\quad}$$

   (c) Using the five steps from (b), what input should you begin with to get a final result of $\frac{14}{27}$?
   Justify your answer.

2. The Fryer Foundation is giving out four types of prizes, valued at $5, $25, $125 and $625.
   (a) The Foundation gives out at least one of each type of prize. If five prizes are given out with a total value of $905, how many of each type of prize is given out? Explain how you got your answer.

   (b) If the Foundation gives out at least one of each type of prize and five prizes in total, determine the other three possible total values it can give out. Explain how you got your answer.

   (c) There are two ways in which the Foundation could give away prizes totalling $880 while making sure to give away at least one and at most six of each prize. Determine the two ways of doing this, and explain how you got your answer.

3. In "The Sun Game", two players take turns placing discs numbered 1 to 9 in the circles on the board. Each number can only be used once. The object of the game is to be the first to place a disc so that the sum of the 3 numbers along a line through the centre circle is 15.
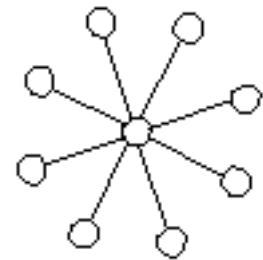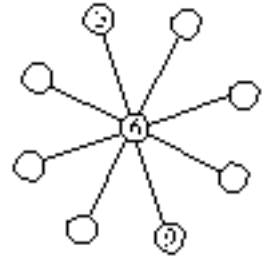

   (a) If Avril places a 5 in the centre circle and then Bob places a 3, explain how Avril can win on her next turn.
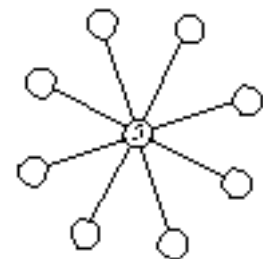
(b) If Avril starts by placing a 5 in the centre circle, show that whatever Bob does on his first turn, Avril can always win on her next turn.
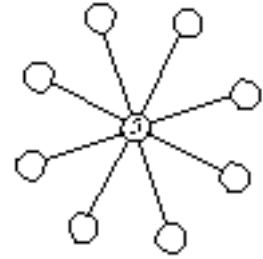
(c) If the game is in the position shown and Bob goes next, show that however Bob plays, Avril can win this game.

4. Triangular numbers can be calculated by counting the dots in the following triangular shapes:


The first triangular number is 1, the second is 3, the third is 6, the fourth is 10, and the nth triangular number equals $1 + 2 + 3 + \cdots + (n-1) + n$.

(a) Calculate the 10th and 24th triangular numbers.

(b) Prove that the sum of any three consecutive triangular numbers is always 1 more than three times the middle of these three triangular numbers.

(c) The 3rd, 6th and 8th triangular numbers (6, 21 and 36) are said to be in arithmetic sequence because the second minus the first equals the third minus the second, ie. $21 - 6 = 36 - 21$. Also, the 8th, 12th and 15th triangular numbers (36, 78 and 120) are in arithmetic sequence. Find three other triangular numbers, each larger than 2004, which are in arithmetic sequence.

2003 *Fryer Contest* (Grade 9)
## Wednesday, April 16, 2003

1. (a) The marks of 32 mathematics students on Test 1 are all multiples of 10 and are shown on the bar graph. What was the average (mean) mark of the 32 students in the class?

Marks on Test 1

Number of Students

Marks (out of 100)
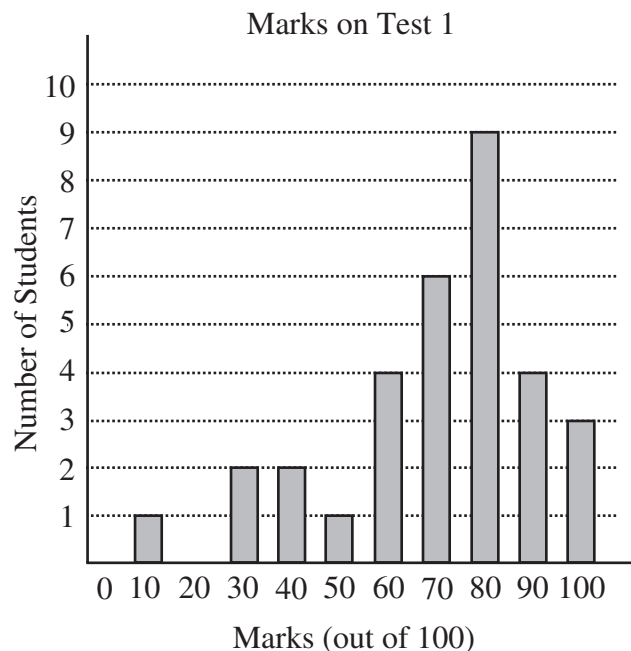
(b) After his first 6 tests, Paul has an average of 86. What will his average be if he scores 100 on his next test?

(c) Later in the year, Mary realizes that she needs a mark of 100 on the next test in order to achieve an average of 90 for all her tests. However, if she gets a mark of 70 on the next test, her average will be 87. After she writes the next test, how many tests will she have written?

2. In a game, Xavier and Yolanda take turns calling out whole numbers. The first number called must be a whole number between and including 1 and 9. Each number called after the first must be a whole number which is 1 to 10 greater than the previous number called.

(a) The first time the game is played, the person who calls the number 15 is the winner. Explain why Xavier has a winning strategy if he goes first and calls 4.

(b) The second time the game is played, the person who calls the number 50 is the winner. If Xavier goes first, how does he guarantee that he will win?

3. In the diagram, $ABCD$ is a square and the coordinates of A and D are as shown.

(a) The point P has coordinates $(10,0)$. Show that the area of triangle PCB is 10.

(b) Point $E(a, 0)$ is on the x-axis such that triangle CBE lies entirely outside square $ABCD$. If the area of the triangle is equal to the area of the square, what is the value of a?

(c) Show that there is no point F on the x-axis for which the area of triangle ABF is equal to the area of square $ABCD$.
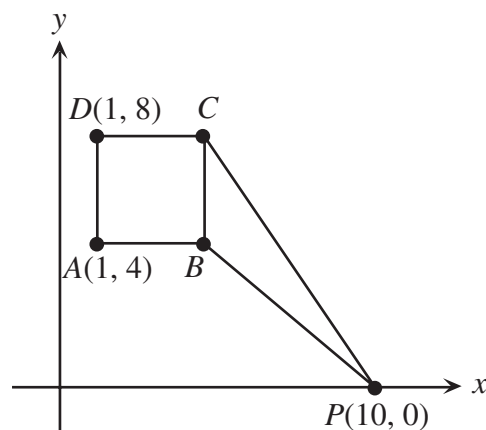
4. For the set of numbers $\{1, 10, 100\}$ we can obtain 7 *distinct* numbers as totals of one or more elements of the set. These totals are $1, 10, 100, 1+10=11, 1+100=101, 10+100=110$, and $1+10+100=111$. The "power-sum" of this set is the *sum* of these totals, in this case, 444.

(a) How many distinct numbers may be obtained as a sum of one or more different numbers from the set $\{1, 10, 100, 1000\}$? Calculate the power-sum for this set.

(b) Determine the power-sum of the set $\{1, 10, 100, 1000, 10\,000, 100\,000, 1\,000\,000\}$.

over ...

Extensions (Attempt these only when you have completed as much as possible of the four main problems.)

Extension to Problem 1:

Mary's teacher records the final marks of the 32 students. The teacher calculates that, for the entire class, the median mark is 80. The teacher also calculates that the difference between the highest and lowest marks is 40 and calculates that the average mark for the entire class is 58. Show that the teacher has made a calculation error.

Extension to Problem 2:

In the game described in (b), the target number was 50. For what different values of the target number is it guaranteed that Yolanda will have a winning strategy if Xavier goes first?

Extension to Problem 3:

G is a point on the line passing through the points $M(0, 8)$ and $N(3, 10)$ such that $\triangle DCG$ lies entirely outside the square. If the area of $\triangle DCG$ is equal to the area of the square, determine the coordinates of G.

Extension to Problem 4:

Consider the set $\{1, 2, 3, 6, 12, 24, 48, 96\}$. How many different totals are now possible if a total is defined as the sum of 1 or more elements of a set?