



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING  
*cemc.uwaterloo.ca*

# Galois Contest

(Grade 10)

Wednesday, April 5, 2023  
(in North America and South America)

Thursday, April 6, 2023  
(outside of North America and South America)



**Time:** 75 minutes

©2023 University of Waterloo

*Do not open this booklet until instructed to do so.*

**Number of questions:** 4

**Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by



- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by



- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks

**WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.**

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example,  $\pi + 1$  and  $1 - \sqrt{2}$  are simplified exact numbers.

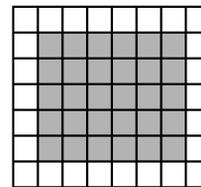
*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

*The name, grade, school and location of some top-scoring students will be published on our website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the  $x$ -intercepts of the graph of an equation like  $y = x^3 - x$ , you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. Jigsaw puzzles often have pieces that are arranged in a rectangular grid of rows and columns, where every cell in the grid represents one piece. The grid has two types of pieces: *edge pieces* which form the outer edge of the grid; and *middle pieces* which form the inside of the grid. In the example shown, there are 7 rows and 8 columns, and the middle pieces are shaded.



- (a)  How many pieces, in total, does a grid with 12 rows and 15 columns have?
- (b)  How many middle pieces does a grid with 6 rows and 4 columns have?
- (c)  If a grid has 14 middle pieces, then it either has  $s$  edge pieces or it has  $t$  edge pieces. Determine the values of  $s$  and  $t$ .
- (d)  A grid with 5 rows and  $c$  columns has the same number of edge pieces as middle pieces. Determine the value of  $c$ .

2. In an *Ing sequence*, the first term is a positive integer and each term after the first is determined in the following way:

- if a term,  $x$ , is odd, the next term is  $x + 3$ , and
- if a term,  $x$ , is even, the next term is  $x + 4$ .

For example, if the first term in an Ing sequence is 13, then the second term is 16, and the third term is 20.

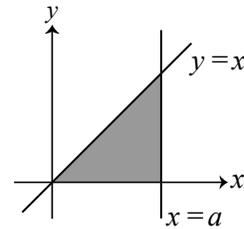
 (a) If the first term in an Ing sequence is 7, what is the fifth term in the sequence?

 (b) If the fifth term in an Ing sequence is 62, what are the two possibilities for the first term?

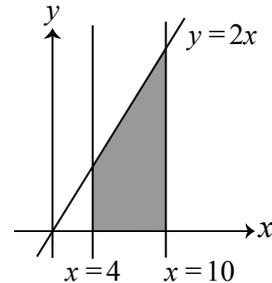
 (c) If the first term in an Ing sequence is 49, determine the terms appearing in the sequence whose values are greater than 318 and less than 330.

 (d) The number 18 appears somewhere in an Ing sequence after the first term. If the first term is the positive integer  $n$ , determine all possible values of  $n$ .

3.  (a) The shaded triangle shown is bounded by the  $x$ -axis, the line  $y = x$ , and the line  $x = a$ , where  $a > 0$ . If the area of this triangle is 32, what is the value of  $a$ ?



 (b) A triangle is bounded by the  $x$ -axis, the line  $y = 2x$ , and the line  $x = 10$ . Diego draws the vertical line  $x = 4$ . This line divides the original triangle into a trapezoid, which is shaded, and a new unshaded triangle, as shown. What is the area of the shaded trapezoid?



 (c) A triangle is bounded by the  $x$ -axis, the line  $y = 3x$ , and the line  $x = 21$ . Alicia draws the vertical line  $x = c$ , where  $0 < c < 21$ . This line divides the original triangle into a trapezoid and a new triangle. If the area of the trapezoid is 8 times the area of the new triangle, determine the value of  $c$ .

 (d) A triangle is bounded by the  $x$ -axis, the line  $y = 4x$ , and the line  $x = 1$ . Ahmed draws his first vertical line at  $x = p$ , where  $0 < p < 1$ . This line divides the area of the original triangle in half. Ahmed then draws a second vertical line at  $x = q$ , where  $0 < q < p$ . This line divides the area of the triangle bounded by the  $x$ -axis, the line  $y = 4x$ , and the line  $x = p$  in half. Ahmed continues this process of drawing vertical lines at decreasing values of  $x$  so that each such line divides the area of the previous triangle in half. If the 12th vertical line that he draws is at  $x = k$ , determine the value of  $k$ .

4. When people gather for a meeting, each person shakes hands with all, some or none of the other people, and never with the same person twice. When a handshake occurs between two people, this is counted as one handshake.



(a) At a meeting of 5 people, Amrita shook hands with exactly 1 person, Bin and Carlos each shook hands with exactly 2 people, Dennis shook hands with exactly 3 people, and Eloise did not shake hands with anyone. How many handshakes took place?



(b) At a meeting of 9 people, each participant said that they shook hands with exactly 3 people. Explain why this is not possible.



(c) At a meeting of 7 people, at least one handshake occurred within each group of 3 people. Determine the minimum possible number of handshakes,  $m$ , that took place at this meeting. A complete solution must include the value of  $m$ , an explanation of why the given conditions can be satisfied with some specific set of  $m$  handshakes, and an explanation of why fewer than  $m$  handshakes does not satisfy the given conditions.



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING  
*cemc.uwaterloo.ca*

**For students...**

Thank you for writing the 2023 Galois Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2023.

Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

**For teachers...**

Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to

- Obtain information about our 2023/2024 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results