



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Hypatia Contest

(Grade 11)

Wednesday, April 12, 2017

(in North America and South America)

Thursday, April 13, 2017

(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 75 minutes

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Do not open this booklet until instructed to do so.

Number of questions: 4

Each question is worth 10 marks

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by



- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by



- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks



WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

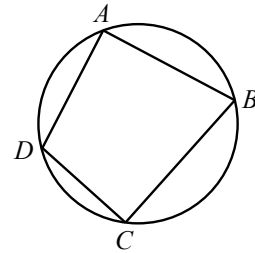
Do not discuss the problems or solutions from this contest online for the next 48 hours.





The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

- Please read the instructions on the front cover of this booklet.
- Write all answers in the answer booklet provided.
- For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
- For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
- Diagrams are *not* drawn to scale. They are intended as aids only.
- While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x -intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
- No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. A *cyclic quadrilateral* is a quadrilateral whose four vertices lie on some circle. In a cyclic quadrilateral, opposite angles add to 180° . In the diagram, $ABCD$ is a cyclic quadrilateral. Therefore, $\angle ABC + \angle ADC = 180^\circ = \angle BAD + \angle BCD$.



-  (a) In Figure A below, $ABCD$ is a cyclic quadrilateral. If $\angle BAD = 88^\circ$, what is the value of u ?
-  (b) In Figure B, $PQRS$ and $STQR$ are cyclic quadrilaterals. If $\angle STQ = 58^\circ$, what is the value of x and what is the value of y ?
-  (c) In Figure C, $JKLM$ is a cyclic quadrilateral with $JK = KL$ and $JL = LM$. If $\angle KJL = 35^\circ$, what is the value of w ?
-  (d) In Figure D, $DEFG$ is a cyclic quadrilateral. FG is extended to H , as shown. If $\angle DEF = z^\circ$, determine the measure of $\angle DGH$ in terms of z .

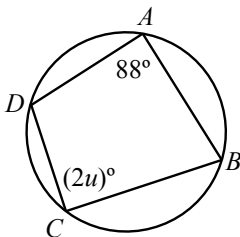


Figure A

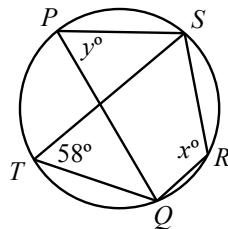


Figure B

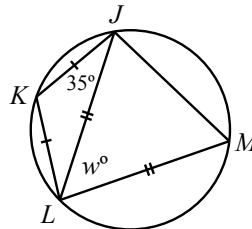


Figure C

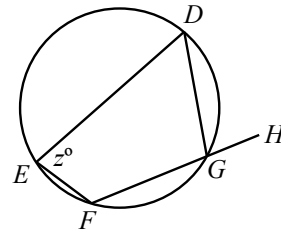










Figure D



2. A list of integers is written in a table, row after row from left to right. Row 1 has the integer 1. Row 2 has the integers 1, 2 and 3. Row n has the consecutive integers beginning at 1 and ending at the n^{th} odd integer. In the table, the 9^{th} integer to be written is 5, and it appears at the end of Row 3. In general, after having completed n rows, a total of n^2 integers have been written.

Row 1	1
Row 2	1 2 3
Row 3	1 2 3 4 5
Row 4	1 2 3 4 5 6 7
\vdots	

-  (a) What is the 25^{th} integer written in the table and in which row does the 25^{th} integer appear?
-  (b) What is the 100^{th} integer written in the table?
-  (c) What is the 2017^{th} integer written in the table?
-  (d) In how many of the first 200 rows does the integer 96 appear?
3.  (a) The line $y = -15$ intersects the parabola with equation $y = -x^2 + 2x$ at two points. What are the coordinates of these two points of intersection?
-  (b) A line intersects the parabola with equation $y = -x^2 - 3x$ at $x = 4$ and at $x = a$. This line intersects the y -axis at $(0, 8)$. Determine the value of a .
-  (c) A line intersects the parabola with equation $y = -x^2 + kx$ at $x = p$ and at $x = q$ with $p \neq q$. Determine the y -intercept of this line.
-  (d) For all $k \neq 0$, the curve $x = \frac{1}{k^3}y^2 + \frac{1}{k}y$ intersects the parabola with equation $y = -x^2 + kx$ at $(0, 0)$ and at a second point T whose coordinates depend on k . All such points T lie on a parabola. Determine the equation of this parabola.

4. A positive integer is called an n -digit zigzag number if
- $3 \leq n \leq 9$,
 - the number's digits are exactly $1, 2, \dots, n$ (without repetition), and
 - for each group of three adjacent digits, either the middle digit is greater than each of the other two digits or the middle digit is less than each of the other two digits.

For example, 52314 is a 5-digit zigzag number but 52143 is not.


-  (a) What is the largest 9-digit zigzag number?
-  (b) Let $G(n, k)$ be the number of n -digit zigzag numbers with first digit k and second digit greater than k . Let $L(n, k)$ be the number of n -digit zigzag numbers with first digit k and second digit less than k .

(i) Show that $G(6, 3) = L(5, 3) + L(5, 4) + L(5, 5)$.

(ii) Show that

$$\begin{aligned} &G(6, 1) + G(6, 2) + G(6, 3) + G(6, 4) + G(6, 5) + G(6, 6) \\ &\text{equals} \end{aligned}$$

$$L(6, 1) + L(6, 2) + L(6, 3) + L(6, 4) + L(6, 5) + L(6, 6).$$

-  (c) Determine the number of 8-digit zigzag numbers.



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