The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Canadian Intermediate Mathematics Contest

Wednesday, November 22, 2017
(in North America and South America)

Thursday, November 23, 2017
(outside of North America and South America)

Time: 2 hours

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.

Do not open this booklet until instructed to do so.

There are two parts to this paper. The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.

PART A

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.** For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name, and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

At the completion of the contest, insert your student information form inside your answer booklet.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on the Web site, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.

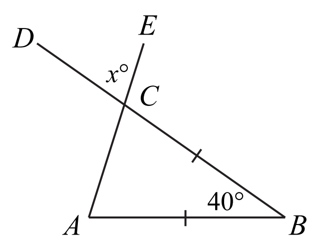
Canadian Intermediate Mathematics Contest

NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.
4. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
5. Diagrams are not drawn to scale. They are intended as aids only.
6. No student may write both the Canadian Senior Mathematics Contest and the Canadian Intermediate Mathematics Contest in the same year.

PART A

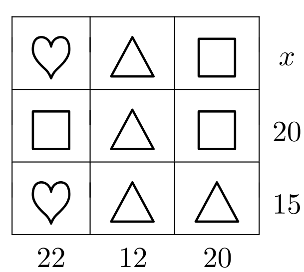
For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

1. In the diagram, BD and AE intersect at C and $AB = BC$. Also, $\angle ABC = 40^\circ$ and $\angle DCE = x^\circ$. What is the value of x?



2. There are 12 different four-digit positive integers that can be made by arranging the digits $1, 2, 7, 7$. These integers are listed from smallest to largest. What is the sum of the 6th and 7th integers in the list?

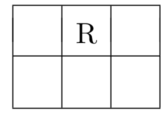
3. In the 3×3 grid shown, each of the three symbols has a different value. The sum of the values of the symbols in each row is given to the right of that row, and the sum of the values of the symbols in each column is given below that column. What is the value of x?



4. Yumi has a flat circular chocolate chip cookie with radius 3 cm. On the top of the cookie, there are k circular chocolate chips, each with radius 0.3 cm. No two chocolate chips overlap and no chocolate chip hangs over the edge of the cookie. For what value of k is exactly $\frac{1}{4}$ of the area of the top of the cookie covered with chocolate chips?

5. Positive integers a, b and c satisfy the equation $\dfrac{31}{72} = \dfrac{a}{8} + \dfrac{b}{9} - c$. What is the smallest possible value of b?

6. In the diagram, six squares form a 2×3 grid. The middle square in the top row is marked with an R. Each of the five remaining squares is to be marked with an R, S or T. In how many ways can the grid be completed so that it includes at least one pair of squares side-by-side in the same row or same column that contain the same letter?



PART B

For each question in Part B, your solution must be well-organized and contain words of explanation or justification. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

1. The first three figures in a pattern are shown below.

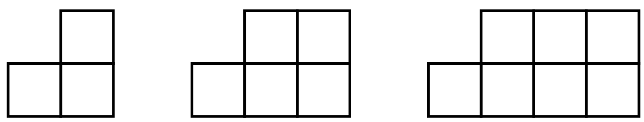


Figure 1 Figure 2 Figure 3

Figure 1 is formed by three identical squares of side length 1 cm arranged in two rows. The perimeter of Figure 1 is 8 cm. Given a figure in the pattern, the next figure is obtained by adding a square of side length 1 cm on the right-hand end of each of the two rows.

(a) How many squares of side length 1 cm are used to form Figure 8?

(b) Determine the perimeter of Figure 12.

(c) Determine the positive integer C for which the perimeter of Figure C is 38 cm.

(d) Determine the positive integer D for which the ratio of the perimeter of Figure 29 to the perimeter of Figure D is equal to $\frac{4}{11}$.

2. The symbol $n!$ represents the product of the positive integers from 1 to n. That is, $n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$. (The symbol $n!$ is read "n factorial".) For example, the value of $4!$ is 24 because $4 \times 3 \times 2 \times 1 = 24$.

(a) Determine the value of $\dfrac{7!}{5!}$.

(b) Determine the positive integer n for which $98! \times 9900 = n!$.

(c) Determine the positive integer m for which $\dfrac{(m+2)!}{m!} = 40\,200$.

(d) Suppose that q is a positive integer and that r is the number for which $(q+2)! - (q+1)! = (q!) \times r$. Show that, for every positive integer q, the number r is an integer which is a perfect square.

3. We call a positive integer *balanced* if
   - it has six digits,
   - each of its six digits is non-zero, and
   - the product of its first three digits is equal to the product of its last three digits.

   For example, 241 181 is balanced since no digit equals zero and $2 \times 4 \times 1 = 1 \times 8 \times 1$.

   (a) Determine, with justification, all balanced positive integers of the form $3b8\,d5f$.

   (b) Determine, with justification, a three-digit positive integer of the form $4bc$ for which there are exactly three balanced positive integers of the form $4bc\,def$.

   (c) For each of $k = 4, 5, 6, 7, 8, 9, 10$, either
   - determine, with justification, an integer abc for which there are exactly k balanced positive integers of the form $abc\,def$, or
   - justify why there does not exist an integer abc for which there are exactly k balanced positive integers of the form $abc\,def$.