The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Canadian Intermediate Mathematics Contest

Wednesday, November 23, 2016
(in North America and South America)

Thursday, November 24, 2016
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 2 hours

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Do not open this booklet until instructed to do so.

There are two parts to this paper. The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.

PART A

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
   For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

At the completion of the contest, insert your student information form inside your answer booklet.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on the Web site, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.

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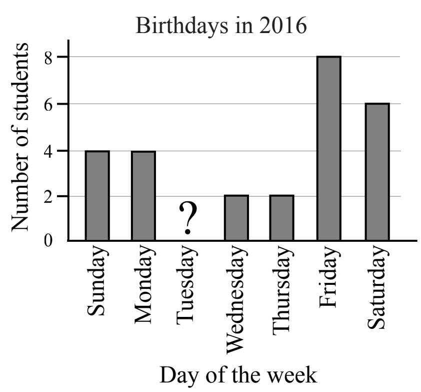
NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.
4. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
5. Diagrams are not drawn to scale. They are intended as aids only.
6. No student may write both the Canadian Senior Mathematics Contest and the Canadian Intermediate Mathematics Contest in the same year.

PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

1. Which of the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{5}{8}$, and $\frac{11}{12}$ is the smallest?

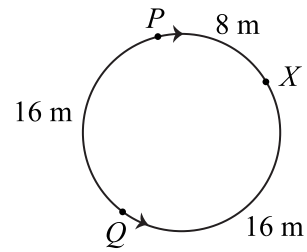
2. The graph to the right shows the number of students in Ms. Gupta's class that have their birthday in 2016 on each day of the week. If 25% of the students in the class have their birthday on a day that begins with the letter "T", how many of the students in the class have their birthday in 2016 on a Tuesday?



3. Pete sets up 12 hurdles for a race that is 600 metres long. The distance between the starting line and the first hurdle is 50 metres. The distance between the last hurdle and the finishing line is 55 metres. The distance between each pair of consecutive hurdles is d metres. What is the value of d?

4. Dina has a calculating machine, labelled f, that takes one number as input and calculates an output. The machine f calculates its output by multiplying its input by 2 and then subtracting 3. For example, if Dina inputs 2.16 into f, the output is 1.32. If Dina inputs a number x into f, she gets a first output which she then inputs back into f to obtain a second output, which is -35. What is the value of x?

5. In the diagram, P and Q start at the positions shown and point X is fixed on the circle. Initially, the shortest distance along the circumference from P to X is 8 m, from Q to X is 16 m, and from P to Q is 16 m, as shown. P and Q move around the circle in opposite directions as indicated by the arrows. P moves at 3 m/s. Q moves at 3.5 m/s. If P and Q begin moving at the same time, after how many seconds do P and Q meet at X for the second time?

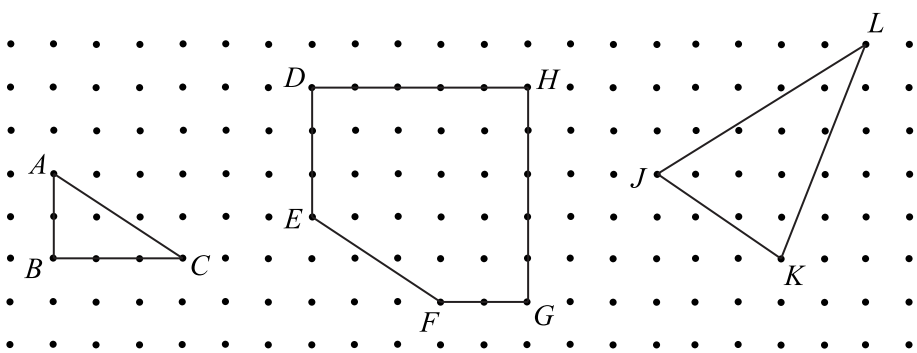


6. A line has equation $y = kx$, where $k \neq 0$ and $k \neq -1$. The line is reflected in the line with equation $x + y = 1$. Determine the slope and the y-intercept of the resulting line, in terms of k.

PART B

For each question in Part B, your solution must be well-organized and contain words of explanation or justification. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

1. In the grid shown, the distance between adjacent dots in one column and adjacent dots in one row is 1 unit.



   (a) Determine the area of $\triangle ABC$.
   (b) Determine the area of figure $DEFGH$.
   (c) Determine the area of $\triangle JKL$.

2. In the list of positive integers $1, 3, 9, 4, 11$ the positive differences between each pair of adjacent integers in the list are $3 - 1 = 2$, $9 - 3 = 6$, $9 - 4 = 5$, and $11 - 4 = 7$. In this example, the smallest positive difference between any two adjacent integers in the list is 2.

   (a) Arrange the integers $1, 2, 3, 4, 5$ so that the smallest positive difference between any two adjacent integers is 2.
   (b) Suppose that the twenty integers $1, 2, 3, \ldots, 18, 19, 20$ are arranged so that the smallest positive difference between any two adjacent integers is N.
      (i) Explain why N cannot be 11 or larger.
      (ii) Find an arrangement with $N = 10$. (Parts (i) and (ii) together prove that the maximum possible value of N is 10.)
   (c) Suppose that the twenty-seven integers $1, 2, 3, \ldots, 25, 26, 27$ are arranged so that the smallest positive difference between any two adjacent integers is N. What is the maximum possible value of N? Prove that your answer is correct.

3. For each positive integer n, the *Murray number of* n is the smallest positive integer M, with $M > n$, for which there exist one or more distinct integers greater than n and less than or equal to M whose product times n is a perfect square.

   For example, the Murray number of 3 is 8 since $3 \times 6 \times 8 = 12^2$ and it can be shown that it is not possible to multiply 3 by one or more distinct integers that are greater than 3 and less than 8 to obtain a perfect square.

   (a) The Murray number of 6 is 12. Show why this is true.

   (b) Determine the Murray number of 8. (No justification is required.)

   (c) Prove that there are infinitely many positive integers n for which n is not a perfect square and the Murray number of n is less than $2n$.

   (d) Prove that, for all positive integers n, the Murray number of n exists and is greater than or equal to $n + 3$.