The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Hypatia Contest

(Grade 11)

Wednesday, April 16, 2014
(in North America and South America)

Thursday, April 17, 2014
(outside of North America and South America)

UNIVERSITY OF WATERLOO WATERLOO MATHEMATICS Deloitte.

Do not open this booklet until instructed to do so.

Time: 75 minutes
Calculators are permitted

Number of questions: 4
Each question is worth 10 marks

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by 
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\ldots$ or $1.41\ldots$, except where otherwise indicated.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our Web site, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

TIPS:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked  , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked  , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.

1. For real numbers a and b with $a \geq 0$ and $b \geq 0$, the operation $\odot$ is defined by

$$a \odot b = \sqrt{a + 4b}.$$

For example, $5 \odot 1 = \sqrt{5 + 4(1)} = \sqrt{9} = 3$.


(a) What is the value of $8 \odot 7$?


(b) If $16 \odot n = 10$, what is the value of n?


(c) Determine the value of $(9 \odot 18) \odot 10$.


(d) With justification, determine all possible values of k such that $k \odot k = k$.

2. Each week, the MathTunes Music Store releases a list of the Top 200 songs. A new song "Recursive Case" is released in time to make it onto the Week 1 list. The song's position, P, on the list in a certain week, w, is given by the equation $P = 3w^2 - 36w + 110$. The week number w is always a positive integer.


(a) What position does the song have on week 1?

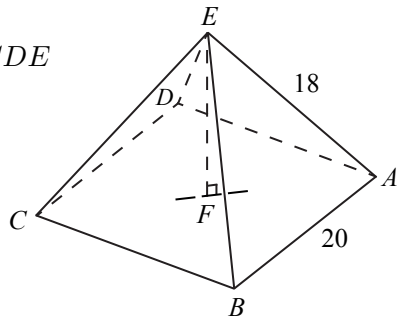

(b) Artists want their song to reach the best position possible. The closer that the position of a song is to position #1, the better the position.

(i) What is the best position that the song "Recursive Case" reaches?

(ii) On what week does this song reach its best position?


(c) What is the last week that "Recursive Case" appears on the Top 200 list?

3. A pyramid $ABCDE$ has a square base $ABCD$ of side length 20. Vertex E lies on the line perpendicular to the base that passes through F, the centre of the base $ABCD$. It is given that $EA = EB = EC = ED = 18$.




(a) Determine the surface area of the pyramid $ABCDE$ including its base.


(b) Determine the height EF of the pyramid.


(c) G and H are the midpoints of ED and EA, respectively. Determine the area of the quadrilateral $BCGH$.

4. The triple of positive integers (x, y, z) is called an Almost Pythagorean Triple (or APT) if $x > 1$ and $y > 1$ and $x^2 + y^2 = z^2 + 1$. For example, (5, 5, 7) is an APT.


(a) Determine the values of y and z so that $(4, y, z)$ is an APT.


(b) Prove that for any triangle whose side lengths form an APT, the area of the triangle is not an integer.


(c) Determine two 5-tuples (b, c, p, q, r) of positive integers with $p \geq 100$ for which $(5t + p, bt + q, ct + r)$ is an APT for all positive integers t.

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For students...

Thank you for writing the 2014 Hypatia Contest!
In 2013, more than 15 000 students from around the world registered to write the Fryer, Galois and Hypatia Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2014.

Visit our website to find

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- Workshops to help you prepare for future contests
- Information about our publications for mathematics enrichment and contest preparation

For teachers...

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