The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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2014 Canadian Team Mathematics Contest

Individual Problems

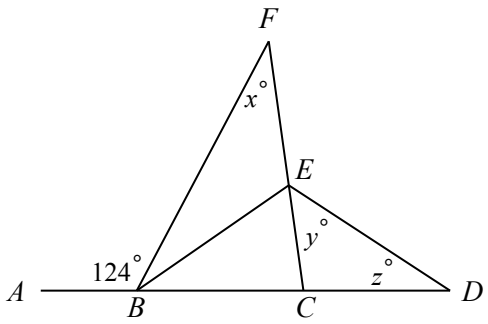
1. The sum of 8 one-digit positive integers is 68. If seven of these integers are equal, determine the other integer.

2. The five-digit positive integer $15AB9$ is a perfect square for some digits A and B. What is the value of $A + B$?

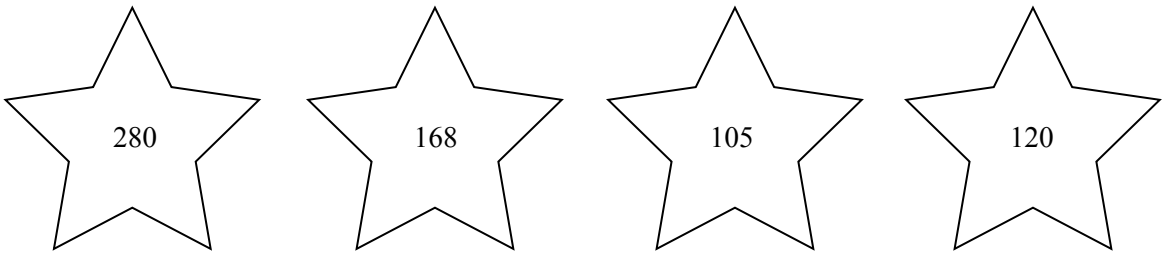
3. If $b \neq 0$, we define $a \heartsuit b = ab - \frac{a}{b}$. For example, $3 \heartsuit 1 = 3(1) - \frac{3}{1} = 0$.
   If $x = 4 \heartsuit 2$ and $y = 2 \heartsuit 2$, what is the value of x^y ?

4. The numbers $36, 27, 42, 32, 28, 31, 23, 17$ are grouped in pairs so that the sum of each pair is the same. Which number is paired with 32?

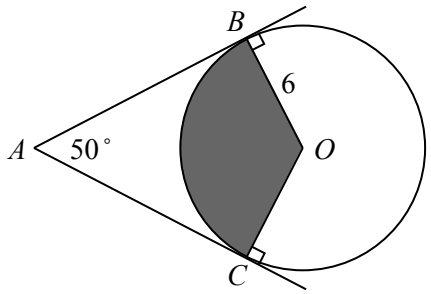
5. In the diagram, points B and C lie on AD and point E lies on CF. The measures of four angles are shown. What is the value of $x + y + z$?



6. In the diagram, a positive integer is hidden behind each star. The integer shown on each star is the product of the integers hidden behind the other three stars. What is the product of all four hidden integers?



7. In the diagram, the circle has centre O and radius 6. Point A is outside the circle and points B and C are on the circle so that AB is perpendicular to BO, AC is perpendicular to CO, and $\angle BAC = 50°$. What is the area of the shaded region?



8. When three students, Al, Betty, and Charlie, compete in a competition, there are 13 possible orders of finish, allowing for the possibility of ties. These possibilities are illustrated in the chart below:

First	A	A	B	B	C	C	ABC	AB	AC	BC	A	B	C
Second	B	C	A	C	A	B		C	B	A	BC	AC	AB
Third	C	B	C	A	B	A							

When four students, David, Erin, Frank, and Greg, compete in a competition, how many possible orders of finish are there, allowing for the possibility of ties?

9. In a geometric sequence with five terms t_1, t_2, t_3, t_4, t_5, each term is positive and $t_1 > t_2$. If the sum of the first two terms is $\frac{15}{2}$ and the sum of the squares of the first two terms is $\frac{153}{4}$, what is the value of t_5?

(A *geometric sequence* is a sequence in which each term after the first is obtained from the previous term by multiplying it by a non-zero constant. For example, 3, 6, 12 is a geometric sequence with three terms.)

10. The diagram shows a rectangular picture frame $ABCD$ made of eight identical trapezoids. The shaded region is where the picture goes. The length, AB, and width, AD, of the frame are both positive integers. The area of each individual trapezoidal piece is a prime number. If the area of the shaded region is less than 2000 square units, what is the maximum possible area of the shaded region?