The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Hypatia Contest

(Grade 11)

Thursday, April 18, 2013
(in North America and South America)

Friday, April 19, 2013
(outside of North America and South America)

Do not open this booklet until instructed to do so.

Time: 75 minutes
Calculators are permitted

Number of questions: 4
Each question is worth 10 marks

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by 
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\ldots$ or $1.41\ldots$, except where otherwise indicated.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our Web site, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

TIPS:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked  , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked  , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.

1. At the JK Mall grand opening, some lucky shoppers are able to participate in a money giveaway. A large box has been filled with many \$5, \$10, \$20, and \$50 bills. The lucky shopper reaches into the box and is allowed to pull out one handful of bills.

   
   (a) Rad pulls out at least two bills of each type and his total sum of money is \$175. What is the total number of bills that Rad pulled out?

   
   (b) Sandy pulls out exactly five bills and notices that she has at least one bill of each type. What are the possible sums of money that Sandy could have?

   
   (c) Lino pulls out six or fewer bills and his total sum of money is \$160. There are exactly four possibilities for the number of each type of bill that Lino could have. Determine these four possibilities.

2. A parabola has equation $y = (x-3)^2 + 1$.

   
   (a) What are the coordinates of the vertex of the parabola?

   
   (b) A new parabola is created by translating the original parabola 3 units to the left and 3 units up. What is the equation of the translated parabola?

   
   (c) Determine the coordinates of the point of intersection of these two parabolas.

   
   (d) The parabola with equation $y = ax^2 + 4$, $a < 0$, touches the parabola with equation $y = (x-3)^2 + 1$ at exactly one point. Determine the value of a.

3. A sequence of m P's and n Q's with $m > n$ is called *non-predictive* if there is some point in the sequence where the number of Q's counted from the left is greater than or equal to the number of P's counted from the left.
For example, if $m = 5$ and $n = 2$ the sequence PPQQPPP is non-predictive because in counting the first four letters from the left, the number of Q's is equal to the number of P's. Also, the sequence QPPPQPP is non-predictive because in counting the first letter from the left, the number of Q's is greater than the number of P's.

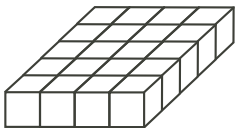
(a) If $m = 7$ and $n = 2$, determine the number of non-predictive sequences that begin with P.

(b) Suppose that $n = 2$. Show that for every $m > 2$, the number of non-predictive sequences that begin with P is equal to the number of non-predictive sequences that begin with Q.

(c) Determine the number of non-predictive sequences with $m = 10$ and $n = 3$.

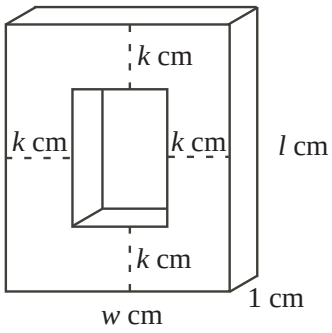
4. 

(a) Twenty cubes, each with edge length 1 cm, are placed together in 4 rows of 5. What is the surface area of this rectangular prism?



(b) A number of cubes, each with edge length 1 cm, are arranged to form a rectangular prism having height 1 cm and a surface area of 180 cm^2. Determine the number of cubes in the rectangular prism.

(c) A number of cubes, each with edge length 1 cm, are arranged to form a rectangular prism having length l cm, width w cm, and thickness 1 cm. A frame is formed by removing a rectangular prism with thickness 1 cm located k cm from each of the sides of the original rectangular prism, as shown. Each of l, w and k is a positive integer. If the frame has surface area 532 cm^2, determine all possible values for l and w such that $l \geq w$.



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For students...

Thank you for writing the 2013 Hypatia Contest!
In 2012, more than 13 000 students from around the world registered to write the Fryer, Galois and Hypatia Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2013.

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