



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING  
*cemc.uwaterloo.ca*

# Galois Contest

(Grade 10)

Thursday, April 18, 2013  
(in North America and South America)

Friday, April 19, 2013  
(outside of North America and South America)

UNIVERSITY OF  
**WATERLOO**

**WATERLOO**  
MATHEMATICS

**Deloitte.**

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*Do not open this booklet until instructed to do so.*

**Time:** 75 minutes

**Calculators are permitted**

**Number of questions:** 4

**Each question is worth 10 marks**

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 

- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by 

- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks

**WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.**

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as  $\pi + 1$  and  $\sqrt{2}$ , etc., rather than as 4.14... or 1.41..., except where otherwise indicated.



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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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*The name, grade, school and location of some top-scoring students will be published on our Web site, <http://www.cemc.uwaterloo.ca>. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

TIPS:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.

1.  (a) Find an equation of the line that passes through the points  $(2, 0)$  and  $(0, 4)$ .  
 (b) Rewrite the equation of the line from part (a) in the form  $\frac{x}{c} + \frac{y}{d} = 1$ , where  $c$  and  $d$  are integers.  
 (c) State the  $x$ -intercept and the  $y$ -intercept of the line  $\frac{x}{3} + \frac{y}{10} = 1$ .  
 (d) Determine the equation of the line that passes through the points  $(8, 0)$  and  $(2, 3)$  written in the form  $\frac{x}{e} + \frac{y}{f} = 1$ , where  $e$  and  $f$  are integers.
2. A thick red candle and a thin green candle are each 100 cm tall. These two candles are lit at the same time. As the candles burn, their heights decrease at constant but different rates. The red candle takes 600 minutes to burn completely. The green candle takes 480 minutes to burn completely.
  -  (a) By how much will the height of the red candle have decreased 180 minutes after being lit?
  -  (b) How many minutes after being lit will the green candle be 80 cm tall?
  -  (c) How much taller will the red candle be than the green candle 60 minutes after they are lit?
  -  (d) How many minutes after being lit will the red candle be 7 cm taller than the green candle?

3. The even positive integers are listed in order and arranged into rows, as shown, and described below.

<u>Row Number</u>				
1	2			
2	4	6		
3	8	10	12	
4	14	16	18	20
				⋮

Each new row includes one more integer than the previous row. The last number in each row is the product of the row number and the next largest integer. For example, the last number in the 4<sup>th</sup> row is  $4 \times 5$ . You may use this fact without proving it.



- (a) List the numbers in the 7th row of the table.



- (b) What are the first and last numbers in the 100th row of the table?

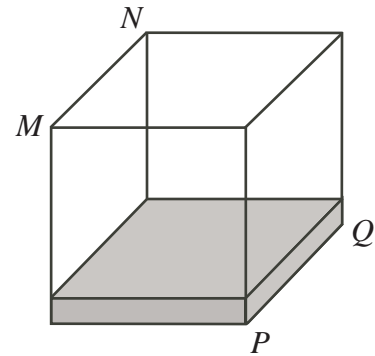


- (c) The last number in row  $r$  is  $L$ . The first number in row  $(r + 2)$  is  $F$ . Determine the smallest possible value for  $r$  such that  $F + L$  is at least 2013.

4. A cube with edge length 9 cm contains a certain amount of water.



- (a) When the cube has one face resting on the ground, the depth of the water is 1 cm, as shown. What is the volume of water in the cube?



- (b) The cube is turned so that one edge,  $PQ$ , is on the ground with the opposite edge,  $MN$ , directly above it. Calculate the depth of the water in the cube.



- (c) The cube is now positioned so that a single corner,  $P$ , is on the ground with the opposite corner,  $N$ , directly above it. To the nearest hundredth of a centimetre, calculate the depth of the water in the cube.



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**For students...**

Thank you for writing the 2013 Galois Contest!  
In 2012, more than 13 000 students from around the world registered to write the Fryer, Galois and Hypatia Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2013.

Visit our website to find

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- Information about our publications for mathematics enrichment and contest preparation

**For teachers...**

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