The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Cayley Contest

(Grade 10)

Thursday, February 21, 2013
(in North America and South America)

Friday, February 22, 2013
(outside of North America and South America)

UNIVERSITY OF WATERLOO WATERLOO MATHEMATICS Deloitte.

Time: 60 minutes
Calculators are permitted
Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{8+4}{8-4}$ is

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

2. The expression $2^3 + 2^2 + 2^1$ is equal to

   (A) 6 **(B)** 10 **(C)** 14 **(D)** 18 **(E)** 22

3. If $x + \sqrt{81} = 25$, then x equals

   (A) 16 **(B)** 56 **(C)** 9 **(D)** 35 **(E)** 4

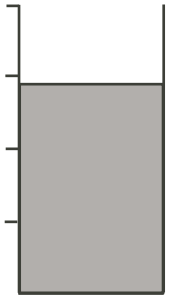
4. How many of the four integers 222, 2222, 22 222, and 222 222 are multiples of 3?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

5. A rectangular field has a length of 20 metres and a width of 5 metres. If its length is increased by 10 m, by how many square metres will its area be increased?

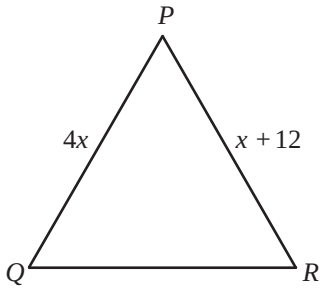
   (A) 10 **(B)** 20 **(C)** 50 **(D)** 75 **(E)** 100

6. A large cylinder can hold 50 L of chocolate milk when full. The tick marks show the division of the cylinder into four parts of equal volume. Which of the following is the best estimate for the volume of chocolate milk in the cylinder as shown?

   (A) 24 L **(B)** 28 L **(C)** 30 L
   (D) 36 L **(E)** 40 L

7. In the diagram, $\triangle PQR$ is an equilateral triangle. If $PQ = 4x$ and $PR = x + 12$, what is the value of x?

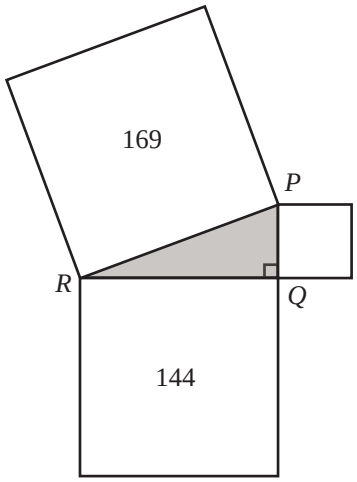
   (A) 48 **(B)** 16 **(C)** 4
   (D) 32 **(E)** 12

8. The symbol $\diamond$ is defined so that $a \diamond b = \dfrac{a+b}{a \times b}$. For example, $2 \diamond 5 = \dfrac{2+5}{2 \times 5} = \dfrac{7}{10}$.

   What is the value of $3 \diamond 6$?

   (A) 9 **(B)** $\dfrac{1}{18}$ **(C)** $\dfrac{1}{6}$ **(D)** 2 **(E)** $\dfrac{1}{2}$

9. In the diagram, $\triangle PQR$ has a right angle at Q. A square is drawn on each side of the triangle. The area of the square on side QR is 144. The area of the square on side PR is 169. What is the area of the square on side PQ?



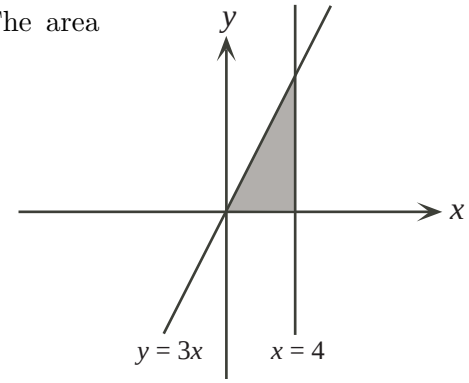
(A) 16 **(B)** 12 **(C)** 13
(D) 36 **(E)** 25

10. Barry has three sisters. The average age of the three sisters is 27. The average age of Barry and his three sisters is 28. What is Barry's age?

(A) 1 **(B)** 30 **(C)** 4 **(D)** 29 **(E)** 31

Part B: Each correct answer is worth 6.

11. The lines with equations $x = 4$ and $y = 3x$ form a triangle with the positive x-axis, as shown. The area of the triangle is



(A) 12 **(B)** 24 **(C)** 36
(D) 48 **(E)** 60

12. If $a(x + b) = 3x + 12$ for all values of x, then $a + b$ equals

(A) 12 **(B)** 15 **(C)** 8 **(D)** 7 **(E)** 13

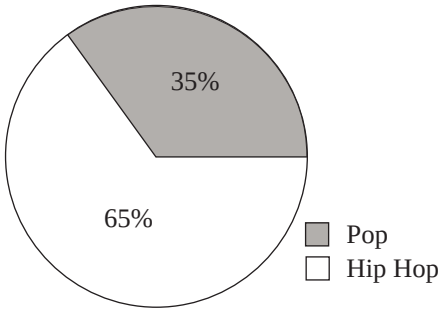
13. An integer x is chosen so that $3x + 1$ is an even integer. Which of the following must be an odd integer?

(A) $x + 3$ **(B)** $x - 3$ **(C)** $2x$ **(D)** $7x + 4$ **(E)** $5x + 3$

14. Integers greater than 1000 are created using the digits 2, 0, 1, 3 exactly once in each integer. What is the difference between the largest and the smallest integers that can be created in this way?

(A) 2187 **(B)** 2333 **(C)** 1980 **(D)** 3209 **(E)** 4233

15. The graph shows styles of music on a playlist. Country music songs are added to the playlist so that 40% of the songs are now Country. If the ratio of Hip Hop songs to Pop songs remains the same, what percentage of the total number of songs are now Hip Hop?

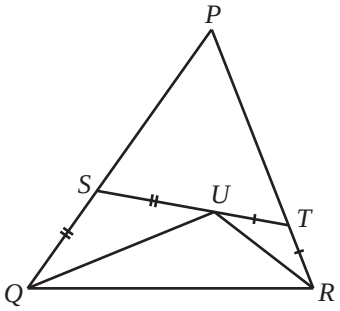


(A) 7 **(B)** 15 **(C)** 21
(D) 35 **(E)** 39

16. When $5^{35} - 6^{21}$ is evaluated, the units (ones) digit is

(A) 1 **(B)** 9 **(C)** 2 **(D)** 5 **(E)** 6

17. In the diagram, $PQ = 19$, $QR = 18$, and $PR = 17$. Point S is on PQ, point T is on PR, and point U is on ST so that $QS = SU$ and $UT = TR$. The perimeter of $\triangle PST$ is equal to

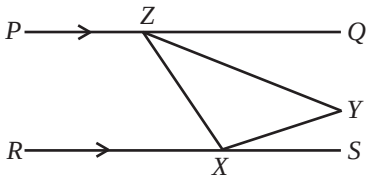


(A) 36 **(B)** 35 **(C)** 37
(D) 34 **(E)** 38

18. A two-digit positive integer x has the property that when 109 is divided by x, the remainder is 4. What is the sum of all such two-digit positive integers x?

(A) 36 **(B)** 56 **(C)** 50 **(D)** 71 **(E)** 35

19. In the diagram, PQ is parallel to RS. Also, Z is on PQ and X is on RS. If Y is located between PQ and RS so that $\angle YXS = 20^\circ$ and $\angle ZYX = 50^\circ$, what is the measure of $\angle QZY$?



(A) 30° **(B)** 20° **(C)** 40°
(D) 50° **(E)** 60°

20. Jack and Jill exercise along the same route. Jill jogs the first half of the route at 6 km/h, runs the remainder of the route at 12 km/h and takes a total time of x hours. Jack walks the first third of the route at 5 km/h, runs the remainder at 15 km/h and takes a total time of y hours. Which of the following is equal to $\frac{x}{y}$?

(A) $\frac{9}{8}$ **(B)** $\frac{7}{5}$ **(C)** $\frac{15}{16}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{10}{9}$

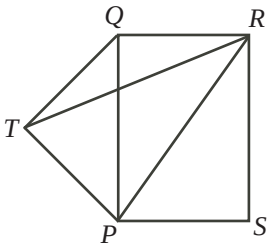
Part C: Each correct answer is worth 8.

21. In the addition shown, the letters X, Y, and Z each represent a different non-zero digit. The digit X is

$$\begin{array}{r} X\ X\ X \\ Y\ Y\ Y \\ +\ Z\ Z\ Z \\ \hline Z\ Y\ Y\ X \end{array}$$

(A) 1 **(B)** 2 **(C)** 7
(D) 8 **(E)** 9

22. In the diagram, $PQRS$ is a rectangle. Point T is outside the rectangle so that $\triangle PTQ$ is an isosceles right-angled triangle with hypotenuse PQ. If $PQ = 4$ and $QR = 3$, then the area of $\triangle PTR$ is

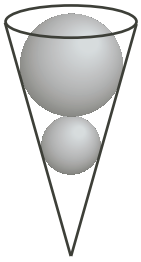


(A) 5 **(B)** 6 **(C)** 7
(D) 8 **(E)** 9

23. One bag contains 2 red marbles and 2 blue marbles. A second bag contains 2 red marbles, 2 blue marbles, and g green marbles, with $g > 0$. For each bag, Maria calculates the probability of randomly drawing two marbles of the same colour in two draws from that bag, without replacement. (Drawing two marbles *without replacement* means drawing two marbles, one after the other, without putting the first marble back into the bag.) If these two probabilities are equal, then the value of g is

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

24. A cone is filled with water. Two solid spheres are placed in the cone as shown in the diagram and water spills out. (The spheres are touching each other, each sphere touches the cone all of the way around, and the top of the top sphere is level with the top of the cone.) The larger sphere has radius twice that of the smaller sphere. If the volume of the water remaining in the cone is 2016π, what is the radius of the smaller sphere? (The volume of a sphere with radius r is $\frac{4}{3}\pi r^3$. The volume of a cone with radius r and height h is $\frac{1}{3}\pi r^2 h$.)



(A) $2\sqrt{2}$ **(B)** 6 **(C)** 8
(D) $6\sqrt{2}$ **(E)** $4\sqrt[3]{2}$

25. A positive integer has k *trailing zeros* if its last k digits are all zero and it has a non-zero digit immediately to the left of these k zeros. For example, the number 1 030 000 has 4 trailing zeros. Define $Z(m)$ to be the number of trailing zeros of the positive integer m. Lloyd is bored one day, so makes a list of the value of $n - Z(n!)$ for each integer n from 100 to 10 000, inclusive. How many integers appear in his list at least three times?
(Note: If n is a positive integer, the symbol $n!$ (read "n factorial") is used to represent the product of the integers from 1 to n. That is, $n! = n(n-1)(n-2)\cdots(3)(2)(1)$. For example, $5! = 5(4)(3)(2)(1)$ or $5! = 120$.)

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

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Thank you for writing the 2013 Cayley Contest!
In 2012, more than 75 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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