



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

www.cemc.uwaterloo.ca

Hypatia Contest

(Grade 11)

Thursday, April 12, 2012

(in North America and South America)

Friday, April 13, 2012

(outside of North America and South America)

UNIVERSITY OF
WATERLOO

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Do not open this booklet until instructed to do so.

Time: 75 minutes

Number of questions: 4

Calculators are permitted

Each question is worth 10 marks

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by



- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by



- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks



WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as 4.14... or 1.41..., except where otherwise indicated.

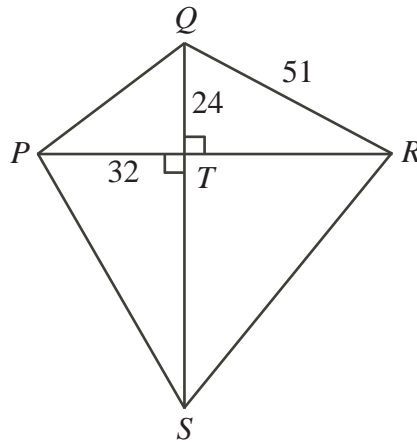
Do not discuss the problems or solutions from this contest online for the next 48 hours.








The name, grade, school and location of some top-scoring students will be published in the FGJ Results on our Web site, <http://www.cemc.uwaterloo.ca>.


TIPS:

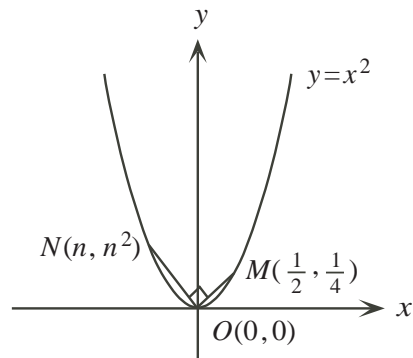
1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.


1. Quadrilateral $PQRS$ is constructed with $QR = 51$, as shown. The diagonals of $PQRS$ intersect at 90° at point T , such that $PT = 32$ and $QT = 24$.

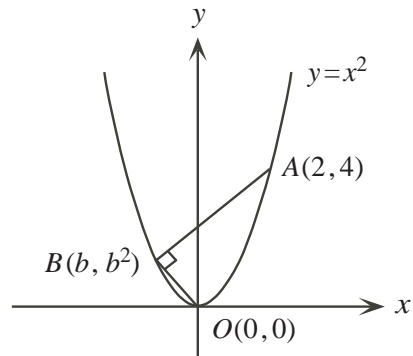



-  (a) Calculate the length of PQ .
 -  (b) Calculate the area of $\triangle PQR$.
 -  (c) If $QS : PR = 12 : 11$, determine the perimeter of quadrilateral $PQRS$.
- 2.
-  (a) Determine the value of $(a + b)^2$, given that $a^2 + b^2 = 24$ and $ab = 6$.
 -  (b) If $(x + y)^2 = 13$ and $x^2 + y^2 = 7$, determine the value of xy .
 -  (c) If $j + k = 6$ and $j^2 + k^2 = 52$, determine the value of jk .
 -  (d) If $m^2 + n^2 = 12$ and $m^4 + n^4 = 136$, determine all possible values of mn .

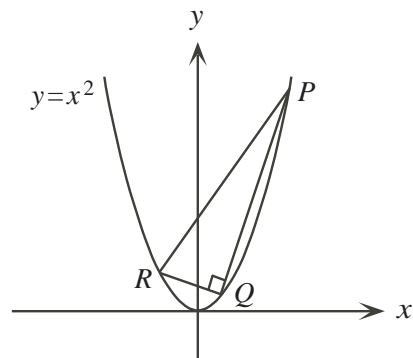
3.  (a) Points $M(\frac{1}{2}, \frac{1}{4})$ and $N(n, n^2)$ lie on the parabola with equation $y = x^2$, as shown. Determine the value of n such that $\angle MON = 90^\circ$.






-  (b) Points $A(2, 4)$ and $B(b, b^2)$ are the endpoints of a chord of the parabola with equation $y = x^2$, as shown. Determine the value of b so that $\angle ABO = 90^\circ$.



-  (c) Right-angled triangle PQR is inscribed in the parabola with equation $y = x^2$, as shown. Points P, Q and R have coordinates $(p, p^2), (q, q^2)$ and (r, r^2) , respectively. If p, q and r are integers, show that $2q + p + r = 0$.



4. The positive divisors of 21 are 1, 3, 7 and 21. Let $S(n)$ be the sum of the positive divisors of the positive integer n . For example, $S(21) = 1 + 3 + 7 + 21 = 32$.

-  (a) If p is an odd prime integer, find the value of p such that $S(2p^2) = 2613$.
-  (b) The consecutive integers 14 and 15 have the property that $S(14) = S(15)$. Determine all pairs of consecutive integers m and n such that $m = 2p$ and $n = 9q$ for prime integers $p, q > 3$, and $S(m) = S(n)$.
-  (c) Determine the number of pairs of distinct prime integers p and q , each less than 30, with the property that $S(p^3q)$ is not divisible by 24.



The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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In 2011, more than 13 000 students from around the world registered to write the Fryer, Galois and Hypatia Contests.

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