



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING

[www.cemc.uwaterloo.ca](http://www.cemc.uwaterloo.ca)

# Galois Contest

(Grade 10)

Thursday, April 12, 2012

(in North America and South America)

Friday, April 13, 2012

(outside of North America and South America)

UNIVERSITY OF  
**WATERLOO**

**WATERLOO  
MATHEMATICS**

THE  
**Great-West Life**  
ASSURANCE COMPANY



 **Canada Life**

STRONGER COMMUNITIES TOGETHER™

Canadian  
Institute of  
Actuaries  Institut  
canadien  
des actuaires

**Deloitte.**

©2012 University of Waterloo

*Do not open this booklet until instructed to do so.*

**Time:** 75 minutes

**Number of questions:** 4

**Calculators are permitted**

**Each question is worth 10 marks**

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by



- worth 2 or 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by



- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks



**WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.**

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as  $\pi + 1$  and  $\sqrt{2}$ , etc., rather than as 4.14... or 1.41..., except where otherwise indicated.

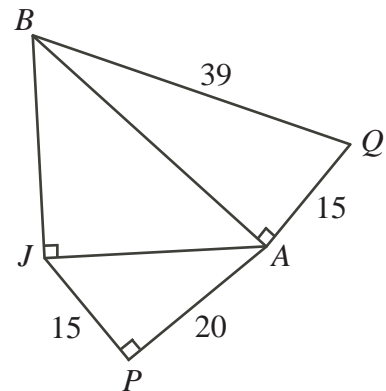
*Do not discuss the problems or solutions from this contest online for the next 48 hours.*




*The name, grade, school and location of some top-scoring students will be published in the FGJ Results on our Web site, <http://www.cemc.uwaterloo.ca>.*

TIPS:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.

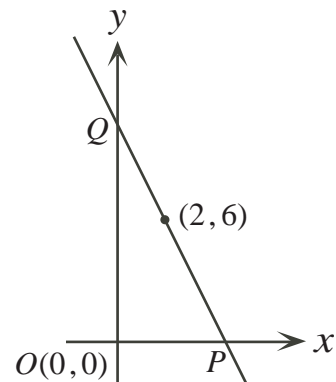
1. Adam and Budan are playing a game of Bocce. Each wants their ball to land closest to the jack ball. The positions of Adam's ball,  $A$ , Budan's ball,  $B$ , and the jack ball,  $J$ , are shown in the diagram.




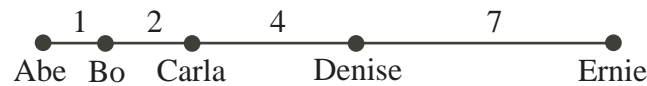
- (a) What is the distance from  $A$  to  $J$ ?
  - (b) What is the distance from  $B$  to  $A$ ?
  - (c) Determine whose ball is closer to the jack ball, Adam's or Budan's.
2.  (a) When the numbers 25, 5 and 29 are taken in pairs and averaged, what are the three averages?
-  (b) When the numbers 2, 6 and  $n$  are taken in pairs and averaged, the averages are 11, 4 and 13. Determine the value of  $n$ .
-  (c) There are three numbers  $a$ ,  $b$  and 2. Each number is added to the average of the other two numbers. The results are 14, 17 and 21. If  $2 < a < b$ , determine the values of  $a$  and  $b$ .

3. The diagram shows one of the infinitely many lines that pass through the point  $(2, 6)$ .

- (a) A line through the point  $(2, 6)$  has slope  $-3$ . Determine the  $x$ - and  $y$ -intercepts of this line.
- (b) Another line through the point  $(2, 6)$  has slope  $m$ . Determine the  $x$ - and  $y$ -intercepts of this line in terms of  $m$ .
- (c) A line through the point  $(2, 6)$  has slope  $m$ , and crosses the positive  $x$ -axis at  $P$  and the positive  $y$ -axis at  $Q$ , as shown. Determine the two values of  $m$  for which  $\triangle POQ$  has an area of 25.



4.  (a) In Town A, five students are standing at different intersections on the same east-west street, as shown. The distances between adjacent intersections are given in kilometres.



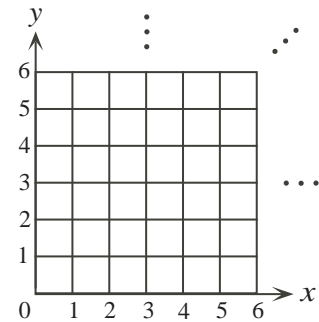
The students agree to meet somewhere on the street such that the total distance travelled by all five students is as small as possible. Where should the students meet?



- (b) In Town B, there is an even number of students. The students are standing at different intersections on a straight north-south street. The students agree to meet somewhere on the street that will make the total distance travelled as small as possible. With justification, determine all possible locations where the students could meet.



- (c) In Town C, the streets run north-south and east-west forming a positive  $xy$ -plane with intersections every 1 km apart, as shown. One hundred students are standing at different intersections. The first 50 students, numbered 1 to 50, stand so that the student numbered  $k$  stands at intersection  $(2^k, k)$ . (For example, student 5 stands at  $(32, 5)$ .) The remaining students, numbered 51 to 100, stand so that the student numbered  $j$  stands at intersection  $(j - 50, 2j - 100)$ . The students can only travel along the streets, and they agree to meet at an intersection that will make the total distance travelled by all students as small as possible. With justification, determine all possible intersections at which the students could meet.





## The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

### For students...

Thank you for writing the 2012 Galois Contest!  
In 2011, more than 13 000 students from around the world registered to write the Fryer, Galois and Hypatia Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2012.

Visit our website to find

- Free copies of past contests
- Workshops to help you prepare for future contests
- Information about our publications for mathematics enrichment and contest preparation

### For teachers...

Visit our website to

- Obtain information about our 2012/2013 contests
- Learn about our face-to-face workshops and our resources
- Find your school contest results
- Subscribe to the Problem of the Week
- Read about our Master of Mathematics for Teachers program

[www.cemc.uwaterloo.ca](http://www.cemc.uwaterloo.ca)