



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

www.cemc.uwaterloo.ca

Euclid Contest

Wednesday, April 11, 2012
(in North America and South America)

Thursday, April 12, 2012
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

**WATERLOO
MATHEMATICS**

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Do not open this booklet until instructed to do so.

Time: $2\frac{1}{2}$ hours

Calculators are permitted, provided they are non-programmable and without graphic displays.

Number of questions: 10

Each question is worth 10 marks

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 

- worth 3 marks each
- full marks given for a correct answer which is placed in the box
- **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by 



- worth the remainder of the 10 marks for the question
- **must be written in the appropriate location** in the answer booklet
- marks awarded for completeness, clarity, and style of presentation
- a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as 4.14... or 1.41..., except where otherwise indicated.


Do not discuss the problems or solutions from this contest online for the next 48 hours.


The name, grade, school and location, and score range of some top-scoring students will be published in the Euclid Results on our Web site, <http://www.cemc.uwaterloo.ca>.


- TIPS:
1. Please read the instructions on the front cover of this booklet.
 2. Write all answers in the answer booklet provided.
 3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
 4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
 5. Diagrams are *not* drawn to scale. They are intended as aids only.


A Note about Bubbling


Please make sure that you have correctly coded your name, date of birth, grade, and sex, on the Student Information Form, and that you have answered the question about eligibility.


1.  (a) John buys 10 bags of apples, each of which contains 20 apples. If he eats 8 apples a day, how many days will it take him to eat the 10 bags of apples?

-  (b) Determine the value of
 $\sin(0^\circ) + \sin(60^\circ) + \sin(120^\circ) + \sin(180^\circ) + \sin(240^\circ) + \sin(300^\circ) + \sin(360^\circ)$


-  (c) A set of integers has a sum of 420, and an average of 60. If one of the integers in the set is 120, what is average of the remaining integers in the set?


2.  (a) If $ax + ay = 4$ and $x + y = 12$, what is the value of a ?

-  (b) If the lines with equations $4x + 6y = 5$ and $6x + ky = 3$ are parallel, what is the value of k ?

-  (c) Determine all pairs (x, y) that satisfy the system of equations

$$\begin{aligned}x + y &= 0 \\x^2 - y &= 2\end{aligned}$$


3.  (a) A 200 g solution consists of water and salt. 25% of the total mass of the solution is salt. How many grams of water need to be added in order to change the solution so that it is 10% salt by mass?

-  (b) The correct formula for converting a Celsius temperature (C) to a Fahrenheit temperature (F) is given by $F = \frac{9}{5}C + 32$.

To approximate the Fahrenheit temperature, Gordie doubles C and then adds 30 to get f .

If $f < F$, the error in the approximation is $F - f$; otherwise, the error in the approximation is $f - F$. (For example, if $F = 68$ and $f = 70$, the error in the approximation is $f - F = 2$.)

Determine the largest possible error in the approximation that Gordie would make when converting Celsius temperatures C with $-20 \leq C \leq 35$.


4.  (a) The horizontal line $y = k$ intersects the parabola with equation $y = 2(x-3)(x-5)$ at points A and B . If the length of line segment AB is 6, what is the value of k ?



- (b) Determine three pairs (a, b) of positive integers for which

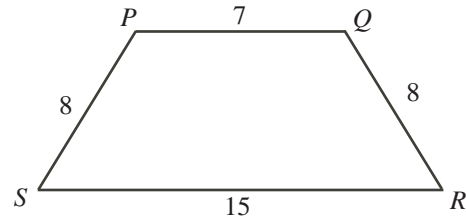
$$(3a + 6a + 9a + 12a + 15a) + (6b + 12b + 18b + 24b + 30b)$$


is a perfect square.

5.  (a) Triangle ABC has vertices $A(0, 5)$, $B(3, 0)$ and $C(8, 3)$. Determine the measure of $\angle ACB$.



- (b) In the diagram, $PQRS$ is an isosceles trapezoid with $PQ = 7$, $PS = QR = 8$, and $SR = 15$. Determine the length of the diagonal PR .




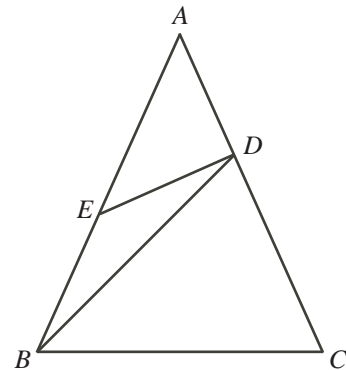
6.  (a) Blaise and Pierre will play 6 games of squash. Since they are equally skilled, each is equally likely to win any given game. (In squash, there are no ties.) The probability that each of them will win 3 of the 6 games is $\frac{5}{16}$. What is the probability that Blaise will win more games than Pierre?



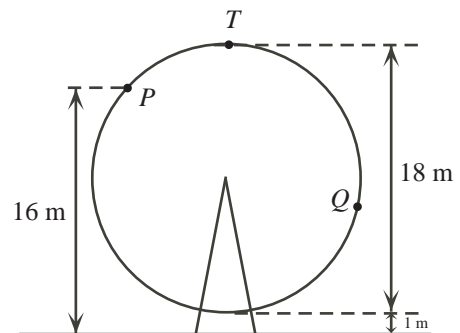
- (b) Determine all real values of x for which


$$3^{x+2} + 2^{x+2} + 2^x = 2^{x+5} + 3^x$$

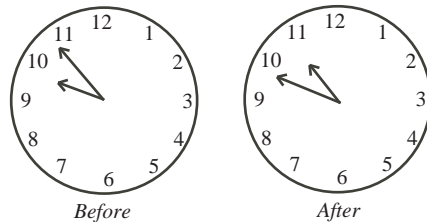
7.  (a) In the diagram, $\triangle ABC$ has $AB = AC$ and $\angle BAC < 60^\circ$. Point D is on AC with $BC = BD$. Point E is on AB with $BE = ED$. If $\angle BAC = \theta$, determine $\angle BED$ in terms of θ .




- (b) In the diagram, the ferris wheel has a diameter of 18 m and rotates at a constant rate. When Kolapo rides the ferris wheel and is at its lowest point, he is 1 m above the ground. When Kolapo is at point P that is 16 m above the ground and is rising, it takes him 4 seconds to reach the highest point, T . He continues to travel for another 8 seconds reaching point Q . Determine Kolapo's height above the ground when he reaches point Q .





8.  (a) On Saturday, Jimmy started painting his toy helicopter between 9:00 a.m. and 10:00 a.m. When he finished between 10:00 a.m. and 11:00 a.m. on the same morning, the hour hand was exactly where the minute hand had been when he started, and the minute hand was exactly where the hour hand had been when he started. Jimmy spent t hours painting. Determine the value of t .

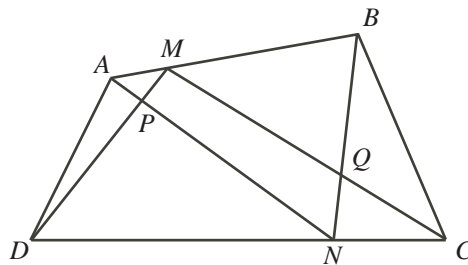



-  (b) Determine all real values of x such that

$$\log_{5x+9}(x^2 + 6x + 9) + \log_{x+3}(5x^2 + 24x + 27) = 4$$

9.  (a) An auditorium has a rectangular array of chairs. There are exactly 14 boys seated in each row and exactly 10 girls seated in each column. If exactly 3 chairs are empty, prove that there are at least 567 chairs in the auditorium.

-  (b) In the diagram, quadrilateral $ABCD$ has points M and N on AB and DC , respectively, with $\frac{AM}{AB} = \frac{NC}{DC}$. Line segments AN and DM intersect at P , while BN and CM intersect at Q . Prove that the area of quadrilateral $PMQN$ equals the sum of the areas of $\triangle APD$ and $\triangle BQC$.



10.  For each positive integer N , an *Eden sequence* from $\{1, 2, 3, \dots, N\}$ is defined to be a sequence that satisfies the following conditions:

- (i) each of its terms is an element of the set of consecutive integers $\{1, 2, 3, \dots, N\}$,
- (ii) the sequence is increasing, and
- (iii) the terms in odd numbered positions are odd and the terms in even numbered positions are even.

For example, the four Eden sequences from $\{1, 2, 3\}$ are

1 3 1, 2 1, 2, 3

- (a) Determine the number of Eden sequences from $\{1, 2, 3, 4, 5\}$.
- (b) For each positive integer N , define $e(N)$ to be the number of Eden sequences from $\{1, 2, 3, \dots, N\}$. If $e(17) = 4180$ and $e(20) = 17710$, determine $e(18)$ and $e(19)$.



The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

For students...

Thank you for writing the 2012 Euclid Contest!

In 2011, more than 16 000 students from around the world registered to write the Euclid Contest.

If you are graduating from secondary school, good luck in your future endeavours! If you will be returning to secondary school next year, encourage your teacher to register you for the 2012 Canadian Senior Mathematics Contest, which will be written in November 2012.

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- Information about our publications for mathematics enrichment and contest preparation

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