The CENTRE for EDUCATION
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2012 Cayley Contest

(Grade 10)

Thursday, February 23, 2012
(in North America and South America)

Friday, February 24, 2012
(outside of North America and South America)

Solutions

1. Simplifying, $\dfrac{5-2}{2+1}=\dfrac{3}{3}=1$.

   Answer: (B)

2. Since the average of three numbers equals 3, then their sum is $3\times 3=9$.
   Therefore, $1+3+x=9$ and so $x=9-4=5$.

   Answer: (B)

3. When the given figure is rotated 90° clockwise, the top edge becomes the right edge, so the two outer shaded triangles are along the right edge of the resulting figure.
   Also, the bottom left shaded triangle moves to the top left.
   Therefore, the resulting figure is the one in (A).

   Answer: (A)

4. Since -1 raised to an even exponent equals 1 and -1 raised to an odd exponent equals -1, then $(-1)^3+(-1)^2+(-1)=-1+1-1=-1$.
   Alternatively, we write

   $$(-1)^3+(-1)^2+(-1)=(-1)(-1)(-1)+(-1)(-1)+(-1)=1(-1)+1-1=-1+1-1=-1$$

   Answer: (D)

5. Since $\sqrt{100-x}=9$, then $100-x=9^2=81$, and so $x=100-81=19$.

   Answer: (E)

6. When 3 bananas are added to the basket, there are 12 apples and 18 bananas in the basket.
   Therefore, the fraction of the fruit in the basket that is bananas is $\frac{18}{12+18}=\frac{18}{30}=\frac{3}{5}$.

   Answer: (C)

7. Since 20% of the students chose pizza and 38% of the students chose Thai food, then the percentage of students that chose Greek food is $100\%-20\%-38\%=42\%$.
   Since there were 150 students surveyed, then $42\%\times 150=\frac{42}{100}\times 150=63$ chose Greek food.

   Answer: (E)

8. Simplifying, $\left(1-\dfrac{1}{3}\right)\left(1-\dfrac{1}{4}\right)\left(1-\dfrac{1}{5}\right)=\left(\dfrac{2}{3}\right)\left(\dfrac{3}{4}\right)\left(\dfrac{4}{5}\right)$.
   We can simplify further by dividing equal numerators and denominators to obtain a final value of $\dfrac{2}{5}$.

   Answer: (A)

9. Since 20 students went skating and 5 students went both skating and skiing, then $20-5=15$ students went skating only.
   Since 9 students went skiing and 5 students went both skating and skiing, then $9-5=4$ students went skiing only.
   The number of students who went skating or skiing or both equals the sum of the number who went skating only, the number who went skiing only, and the number who went both skating and skiing, or $15+4+5=24$.
   Therefore, $30-24=6$ students did not go skating or skiing.

   Answer: (B)

10. The original prism has four faces that are 4 by 2 rectangles, and two faces that are 2 by 2 rectangles. Thus, the surface area of the original prism is $4(4 \cdot 2) + 2(2 \cdot 2) = 32 + 8 = 40$.
When a 1 by 1 by 1 cube is cut out, a 1 by 1 square is removed from each of three faces of the prism, but three new 1 by 1 squares become part of the surface area.
In other words, there is no change to the total surface area.
Therefore, the surface area of the new solid is also 40.

Answer: (C)

11. During a 3 hour shift, Matilda will deliver $3 \times 30 = 90$ newspapers.
Therefore, she earns a total of $3 \times \$6.00 + 90 \times \$0.25 = \$18.00 + \$22.50 = \$40.50$ during her 3 hour shift.

Answer: (A)

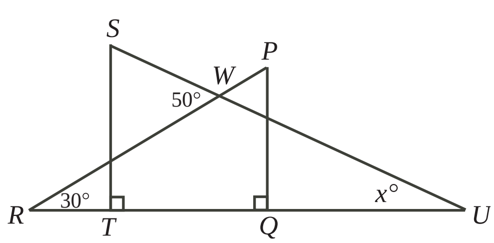
12. Since (p, q) lies on the line $y = \frac{2}{5}x$, then $q = \frac{2}{5}p$.
The given rectangle has two sides on the axes, so has width p and height q.
Therefore, the area of the rectangle equals $pq = p \cdot \frac{2}{5}p = \frac{2}{5}p^2$.
Since we are told that the area of the rectangle is 90, then $\frac{2}{5}p^2 = 90$ or $p^2 = \frac{5}{2}(90) = 225$.
Since $p > 0$, then $p = \sqrt{225} = 15$.

Answer: (D)

13. If N is divisible by both 5 and 11, then N is divisible by $5 \times 11 = 55$.
This is because 5 and 11 have no common divisor larger than 1.
Therefore, we are looking for a multiple of 55 between 400 and 600 that is odd.
One way to find such a multiple is to start with a known multiple of 55, such as 550, which is even.
We can add or subtract 55 from this multiple and still obtain multiples of 55.
Note that $550 + 55 = 605$, which is too large.
Now $550 - 55 = 495$ which is in the correct range and is odd.
Since we are told that there is only such such integer, then it must be the case that $N = 495$.
The sum of the digits of N is $4 + 9 + 5 = 18$.

Answer: (E)

14. We label the point of intersection of RP and SU as W.



Now $\angle SWR$ is an exterior angle for $\triangle RWU$.
Therefore, $\angle SWR = \angle WRU + \angle WUR$, and so $50^\circ = 30^\circ + x^\circ$ or $x = 50 - 30 = 20$.
Alternatively, we could see that $\angle RWU = 180^\circ - \angle SWR = 180^\circ - 50^\circ = 130^\circ$.
Since the angles in $\triangle RWU$ add to 180°, then $30^\circ + 130^\circ + x^\circ = 180^\circ$, or $x = 180 - 130 - 30 = 20$.

Answer: (B)

15. Since the radius of the larger circle is 9, then $OQ = 9$ and the area of the larger circle is $\pi 9^2 = 81\pi$.
Since $OP : PQ = 1 : 2$ and $OQ = 9$, then $OP = \frac{1}{3}OQ = 3$.
Thus, the radius of the smaller circle is 3 and so the area of the smaller circle is $\pi 3^2 = 9\pi$.
The area of the shaded region equals the area of the large circle minus the area of the small circle, or $81\pi - 9\pi = 72\pi$.

ANSWER: (D)

16. From the table of values, when $x = 0$, $y = 8$, and so $8 = a \cdot 0^2 + b \cdot 0 + c$ or $c = 8$.
From the table of values, when $x = 1$, $y = 9$, and so $9 = a \cdot 1^2 + b \cdot 1 + c$ or $a + b + c = 9$.
Since $a + b + c = 9$ and $c = 8$, then $a + b + 8 = 9$ or $a + b = 1$.

ANSWER: (B)

17. Let L be the length of the string.
If x is the length of the shortest piece, then since each of the other pieces is twice the length of the next smaller piece, then the lengths of the remaining pieces are $2x$, $4x$, and $8x$.
Since these four pieces make up the full length of the string, then $x + 2x + 4x + 8x = L$ or $15x = L$ and so $x = \frac{1}{15}L$.
Thus, the longest piece has length $8x = \frac{8}{15}L$, which is $\frac{8}{15}$ of the length of the string.

ANSWER: (A)

18. *Solution 1*
After one of the six integers is erased, there are five integers remaining which add to 2012.
Since the original six integers are consecutive, then we can treat them as roughly equal.
Since there are five roughly equal integers that add to 2012, then each is roughly equal to $\frac{2012}{5}$, which is roughly 400.
We finish our solution by trial and error.
Suppose that the original six integers were 400, 401, 402, 403, 404, 405.
The sum of these integers is 2415. If one of the integers is to be removed to obtain a total of 2012, then the integer removed must be $2415 - 2012 = 403$.
Is there another possible answer?
Suppose that the original six integers were larger, say 401, 402, 403, 404, 405, 406. In this case, the smallest that the sum of five of these could be is $401 + 402 + 403 + 404 + 405 = 2015$, which is too large for the given sum. Any larger set of integers only makes the smallest possible sum of five integers larger.
Suppose that the original six integers were smaller, say 399, 400, 401, 402, 403, 404. In this case, the largest that the sum of five of these could be is $400 + 401 + 402 + 403 + 404 = 2010$, which is too small for the given sum. Any smaller set of integers only makes the largest possible sum of five integers smaller.
Therefore, the possibility found above is the only possibility, and so the sum of the digits of the integer that was erased is $4 + 0 + 3 = 7$.
(Note that, since this is a multiple choice problem, once we had found an answer that works, it must be the correct answer.)

Solution 2
Suppose that the original six integers are x, $x + 1$, $x + 2$, $x + 3$, $x + 4$, and $x + 5$.
Suppose also that the integer that was erased is $x + a$, where a is 0, 1, 2, 3, 4, or 5.

The sum of the integers left is $(x + (x + 1) + (x + 2) + (x + 3) + (x + 4) + (x + 5)) - (x + a)$. Therefore,

$$\begin{aligned} (x + (x + 1) + (x + 2) + (x + 3) + (x + 4) + (x + 5)) - (x + a) &= 2012 \\ (6x + 15) - (x + a) &= 2012 \\ 5x + 15 &= 2012 + a \\ 5(x + 3) &= 2012 + a \end{aligned}$$

Since the left side is an integer that is divisible by 5, then the right side is an integer that is divisible by 5.
Since a is 0, 1, 2, 3, 4, or 5 and $2012 + a$ is divisible by 5, then a must equal 3.
Thus, $5(x + 3) = 2015$ or $x + 3 = 403$ and so $x = 400$.
Finally, the integer that was erased is $x+a = 400+3 = 403$. The sum of its digits is $4+0+3 = 7$.

Answer: (C)

19. Suppose that the sum of the four integers along each straight line equals S.
Then $S = 9 + p + q + 7 = 3 + p + u + 15 = 3 + q + r + 11 = 9 + u + s + 11 = 15 + s + r + 7$.
Thus,

$$\begin{aligned} 5S &= (9 + p + q + 7) + (3 + p + u + 15) + (3 + q + r + 11) + (9 + u + s + 11) + (15 + s + r + 7) \\ &= 2p + 2q + 2r + 2s + 2u + 90 \end{aligned}$$

Since p, q, r, s, and u are the numbers 19, 21, 23, 25, and 27 in some order, then

$$p + q + r + s + u = 19 + 21 + 23 + 25 + 27 = 115$$

and so $5S = 2(115) + 90 = 320$ or $S = 64$.
Since $S = 64$, then $3 + p + u + 15 = 64$ or $p + u = 46$.
Since $S = 64$, then $15 + s + r + 7 = 64$ or $s + r = 42$.
Therefore, $q = (p + q + r + s + u) - (p + u) - (s + r) = 115 - 46 - 42 = 27$.

Answer: (D)

20. In order to find N, which is the smallest possible integer whose digits have a fixed product, we must first find the minimum possible number of digits with this product. (This is because if the integer a has more digits than the integer b, then $a > b$.)
Once we have determined the digits that form N, then the integer N itself is formed by writing the digits in increasing order. (Given a fixed set of digits, the leading digit of N will contribute to the largest place value, and so should be the smallest digit. The next largest place value should get the next smallest digit, and so on.)
Note that the digits of N cannot include 0, or else the product of its digits would be 0.
Also, the digits of N cannot include 1, otherwise we could remove the 1s and obtain an integer with fewer digits (thus, a smaller integer) with the same product of digits.

Since the product of the digits of N is 2700, we find the prime factorization of 2700 to help us determine what the digits are:

$$2700 = 27 \times 100 = 3^3 \times 10^2 = 3^3 \times 2^2 \times 5^2$$

In order for a non-zero digit to have a factor of 5, then the digit must equal 5.
Since 2700 has two factors of 5, then the digits of N includes two 5s.

The remaining digits have a product of $3^3 \times 2^2 = 108$.
Therefore, we must try to find a combination of the smallest number of possible digits whose product is 108.
We cannot have 1 digit with a product of 108. We also cannot have a 2 digits with a product of 108, as the product of 2 digits is at most $9 \times 9 = 81$.
We can have a product of 3 digits with a product of 108 (for example, $2 \times 6 \times 9$ or $3 \times 6 \times 6$).
Therefore, the number N has 5 digits (two 5s and three other digits with a product of 108).
In order for N to be as small as possible, its leading digit (that is, its ten thousands digit) must be as small as possible. Recall that N cannot include the digit 1.
The next smallest possible leading digit is 2. In this case, 2 must be one of the three digits whose product is 108. Thus, the remaining two of these three digits have a product of $108 \div 2 = 54$, and so must be 6 and 9.
Therefore, the digits of N must be 2, 6, 9, 5, 5. The smallest possible number formed by these digits is when the digits are placed in increasing order, and so $N = 25\,569$.
The sum of the digits of N is $2 + 5 + 5 + 6 + 9 = 27$.

Answer: (E)

21. Since $x + xy = 391$, then $x(1 + y) = 391$.
We note that $391 = 17 \cdot 23$.
Since 17 and 23 are both prime, then if 391 is written as the product of two positive integers, it must be 1×391 or 17×23 or 23×17 or 391×1.
Matching x and $1+y$ to these possible factors, we obtain $(x, y) = (1, 390)$ or $(17, 22)$ or $(23, 16)$ or $(391, 0)$.
Since y is a positive integer, the fourth pair is not possible.
Since $x > y$, the first two pairs are not possible.
Therefore, $(x, y) = (23, 16)$ and so $x + y = 39$.

Answer: (B)

22. Since the five monkeys are randomly numbered, then the probability that any given monkey is numbered Monkey 1 is $\frac{1}{5}$. There are five possibilities to consider.

- Case 1: If the monkey in seat P is numbered Monkey 1, then it stays in its seat, so the monkey in seat R cannot move to seat P.
- Case 2: If the monkey in seat Q is numbered Monkey 1, then after the monkeys have moved, Monkey 2 will be in seat R, Monkey 3 in seat S, Monkey 4 in seat T, and Monkey 5 in seat P. Thus, if Monkey 1 is in seat Q, then the monkey in seat R moves to seat P if it was numbered Monkey 5.
- Case 3: If the monkey in seat R is numbered Monkey 1, then it stays in its seat and so cannot move to seat P.
- Case 4: If the monkey in seat S is numbered Monkey 1, then after the monkeys have moved, Monkey 2 will be in seat T, Monkey 3 in seat P, Monkey 4 in seat Q, and Monkey 5 in seat R. Thus, if Monkey 1 is in seat S, then the monkey in seat R moves to seat P if it was numbered Monkey 3.
- Case 5: If the monkey in seat T is numbered Monkey 1, then after the monkeys have moved, Monkey 2 will be in seat P, Monkey 3 in seat Q, Monkey 4 in seat R, and Monkey 5 in seat S. Thus, if Monkey 1 is in seat T, then the monkey in seat R moves to seat P if it was numbered Monkey 2.

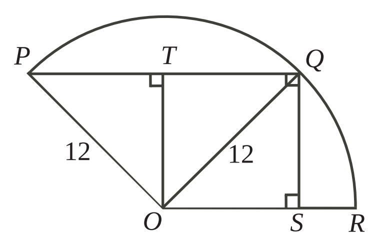
The possible cases that we have to consider are Cases 2, 4 and 5.

- From Case 2, what is the probability that the monkey in seat Q is numbered Monkey 1 and the monkey in seat R is numbered Monkey 5? We know that the probability that the monkey in seat Q is numbered Monkey 1 is $\frac{1}{5}$. Given this and the fact that the remaining four monkeys are numbered randomly, the probability is $\frac{1}{4}$ that the monkey in seat R is numbered Monkey 5. Therefore, the probability of this combined event happening is $\frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20}$.
- Similarly, from Case 4 the probability that the monkey in seat S is numbered Monkey 1 and the monkey in seat R is numbered Monkey 3 is also $\frac{1}{20}$.
- Also, from Case 5 the probability that the monkey in seat T is numbered Monkey 1 and the monkey in seat R is numbered Monkey 2 is $\frac{1}{20}$.

Therefore, the probability that the monkey who was in seat R moves to seat P is $3 \cdot \frac{1}{20} = \frac{3}{20}$.

Answer: (C)

23. Join O to Q and draw a perpendicular from O to T on PQ.
Since the radius of the circle is 12, then $OP = OQ = 12$.
Consider $\triangle OTP$ and $\triangle OTQ$. Each is right-angled, they share side OT, and they have hypotenuses (OP and OQ) of equal length. Therefore, these triangles are congruent.
Consider quadrilateral $TQSO$. Since the quadrilateral has three right angles, then it must be a rectangle so its fourth angle, $\angle TOS$, is 90°.



Thus, $\angle TOP = 135^\circ - 90^\circ = 45^\circ$.
Since the angles in $\triangle OTP$ add to 180°, then $\angle OPT = 180^\circ - 90^\circ - 45^\circ = 45^\circ$.
Therefore, $\triangle OTP$ is isosceles and right-angled with hypotenuse 12.
Since $\triangle OTQ$ is congruent to $\triangle OTP$, it is also isosceles and right-angled with hypotenuse 12.
Since $\angle TOP = \angle TOQ = 45^\circ$, then $\angle QOS = 135^\circ - 45^\circ - 45^\circ = 45^\circ$, which tells us that $\triangle OQS$, which is right-angled, has one 45° angle and so must have a second. Therefore, $\triangle OQS$ is also isosceles and right-angled, and also has hypotenuse $OQ = 12$.
So $\triangle OQS$ is congruent to $\triangle OTQ$.
Therefore, the area of trapezoid $OPQS$ equals three times the area of an isosceles right-angled triangle with hypotenuse 12.
We calculate the area of $\triangle OTP$, which is one of these triangles.
Suppose that $OT = TP = a$.
Since $\triangle OTP$ is right-angled and isosceles, then $OP = \sqrt{2}a$.
(We can see this by using the Pythagorean Theorem to obtain

$$OP = \sqrt{OT^2 + TP^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a$$

since $a > 0$.)
Since $OP = 12$, then $\sqrt{2}a = 12$ or $a = \dfrac{12}{\sqrt{2}}$.
The area of $\triangle OTP$ is $\dfrac{1}{2}(OT)(TP) = \dfrac{1}{2}a^2 = \dfrac{1}{2}\left(\dfrac{12}{\sqrt{2}}\right)^2 = \dfrac{1}{2}\left(\dfrac{144}{2}\right) = 36$.
Thus, the area of trapezoid $OPQS$ is $3 \times 36 = 108$.

Answer: (C)

24. We solve this problem by first determining the possible structures of the exchange within the book club. Then, we count the number of ways the friends can be fit into the structures.
Suppose that the six friends exchange books as described.
Consider one person, who we call A.
A must give a book to a second person, who we call B ($A \to B$).
B cannot give a book back to A, so must give a book to a third person, who we call C ($A \to B \to C$).
C cannot give a book to B since B already receives a book from A. Therefore, C can give a book to A ($A \to B \to C \to A$) or C can give a book to a fourth person, who we call D ($A \to B \to C \to D$).

- In the first case ($A \to B \to C \to A$), each of A, B and C already both give and receive a book. Therefore, the fourth person, who we call D, must give a book to a fifth person, who we call E ($A \to B \to C \to A$; $D \to E$).
  E cannot give a book back to D and cannot give to any of A, B or C, so must give a book to the sixth (and final) person, who we call F ($A \to B \to C \to A$; $D \to E \to F$).
  The only person that F can give a book to is D, since everyone else already is receiving a book. This completes this case as ($A \to B \to C \to A$; $D \to E \to F \to D$).
- The other option from above is $A \to B \to C \to D$.
  Here, D cannot give a book to B or C who each already receive a book. Thus, D gives a book to A or to one of the two remaining people.
  If D gives a book to A, then each of A, B, C and D both gives and receives, so the final two people are left to exchange books with each other, which is not possible.
  Therefore, D gives a book to one of the remaining people, who we call E (giving $A \to B \to C \to D \to E$).
  Only two people have not received a book now: A and the sixth person, who we call F.
  E must give a book to F, otherwise F does not give or receive a book at all. Thus, F must give a book to A.
  This gives the structure $A \to B \to C \to D \to E \to F \to A$.

Therefore, there are two possible structures for the exchanges: $A \to B \to C \to A$ with $D \to E \to F \to D$, and $A \to B \to C \to D \to E \to F \to A$.
(We can think of the first structure as two cycles of three people, and the second as one cycle of six people.)

Suppose the six friends are P, Q, R, S, T, and U. (We could call them Peter, Quinn, Rad, Steve, Troy, and Ursula, but will use abbreviations to save space.) We must count the number of ways that these six friends can be assigned to the positions A through F in the two structures above. We have to be careful because assigning

$$\begin{array}{ccccccc} A & \to & B & \to & C & \to & A \\ P & & Q & & R & & P \end{array} \qquad \begin{array}{ccccccc} D & \to & E & \to & F & \to & D \\ S & & T & & U & & S \end{array}$$

in the first structure is the same as assigning

$$\begin{array}{ccccccc} A & \to & B & \to & C & \to & A \\ U & & S & & T & & U \end{array} \qquad \begin{array}{ccccccc} D & \to & E & \to & F & \to & D \\ Q & & P & & R & & Q \end{array}$$

but is different from assigning

$$\begin{array}{ccccccc} A & \rightarrow & B & \rightarrow & C & \rightarrow & A \\ P & & R & & Q & & P \end{array} \qquad \begin{array}{ccccccc} D & \rightarrow & E & \rightarrow & F & \rightarrow & D \\ S & & T & & U & & S \end{array}$$

- Case 1: $A \rightarrow B \rightarrow C \rightarrow A$ with $D \rightarrow E \rightarrow F \rightarrow D$
  Because this case consists of two cycles of three people each, each person serves exactly the same function, so we can assign P to position A.
  There are then 5 possible friends for position B (all but P).
  For each of these, there are 4 possible friends for position C (all put P and the friend in position B). This completes the first cycle.
  Now, the remaining three friends must complete the second cycle.
  Suppose without loss of generality that these three friends are S, T and U.
  Since each of the three positions in the second cycle serves exactly the same function, we can assign S to position D.
  There are then 2 possible friends for position E (either T or U).
  For each of these, there is 1 possible friend left to go in position F.
  This gives $5 \times 4 \times 2 \times 1 = 40$ ways of assigning the friends to this structure.
- Case 2: $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow A$
  Because this case is a single cycle of six people, each person serves exactly the same function, so we can assign P to position A.
  There are then 5 possible friends for position B (all but P).
  For each of these, there are 4 possible friends for position C (all put P and the friend in position B).
  Similarly, there are 3 possibilities for position D, 2 for position E, and 1 for position F.
  This gives $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways of assigning the friends to this structure.

Therefore, there are $40 + 120 = 160$ ways in total in which the books can be exchanged.

ANSWER: (E)

25. Let $S(n)$ represent the sum of the digits of n and let $S(2n)$ represent the sum of the digits of $2n$. In the table below, we make a claim about how each digit of n contributes to $S(2n)$. We use the data in the table to answer the question, following which we justify the data in the table:

Digit in n	$2\times$Digit	Contribution to $S(2n)$
0	0	0
1	2	2
2	4	4
3	6	6
4	8	8
5	10	$1 + 0 = 1$
6	12	$1 + 2 = 3$
7	14	$1 + 4 = 5$
8	16	$1 + 6 = 7$

We know that the digits of n include no 9s, exactly four 8s, exactly three 7s, and exactly two 6s. These digits contribute $4 \cdot 8 + 3 \cdot 7 + 2 \cdot 6 = 65$ to $S(n)$, leaving a sum of $104 - 65 = 39$ for the remaining digits.
Suppose that n includes m 5s. These 5s contribute $5m$ to $S(n)$, so the remaining $39 - 5m$ come from the digits 0 to 4.

Consider now the digits of $2n$.
The table above shows that the 9s, 8s, 7s and 6s each contribute 9, 7, 5 and 3, respectively, to $S(2n)$. Since the digits of n include no 9s, four 8s, three 7s, and two 6s, then these digits contribute $4 \cdot 7 + 3 \cdot 5 + 2 \cdot 3 = 49$ to $S(2n)$.
Each 5 in n contributes 1 to $S(2n)$, so the m 5s in n contribute $m \cdot 1 = m$ to $S(2n)$.
The digits 0 to 4 each contribute twice as much to $S(2n)$ as they do to $S(n)$, so the sum of the their contributions to $S(2n)$ is twice the sum of their contributions to $S(n)$.
Since the sum of their contributions to $S(n)$ is $39 - 5m$, then the sum of their contributions to $S(2n)$ is $2(39 - 5m)$.
Since $S(2n) = 100$, then $49 + m + 2(39 - 5m) = 100$.
Therefore, $127 - 9m = 100$ and so $9m = 27$ or $m = 3$.
That is, the digits of n include three 5s.

We must still justify the data in the table above.
Suppose that n ends with the digits $dcba$. That is, $n = \cdots dcba$.
Then we can write $n = \cdots + 1000d + 100c + 10b + a$.
Then $2n = \cdots + 1000(2d) + 100(2c) + 10(2b) + (2a)$. The difficulty in determining the digits of $2n$ is that each of $2d$, $2c$, $2b$ and $2a$ may not be a single digit.
We use the notation $u(2a)$ and $t(2a)$ to represent the units digit and tens digit of $2a$, respectively.
Note that $u(2a)$ is one of 0, 2, 4, 6, or 8, and $t(2a)$ is 0 or 1.
We define $u(2b), t(2b), u(2c), t(2c), u(2d), t(2d)$ similarly.
Note that $2a = 10 \cdot t(2a) + u(2a)$ and $2b = 10 \cdot t(2b) + u(2b)$ and $2c = 10 \cdot t(2c) + u(2c)$ and $2d = 10 \cdot t(2d) + u(2d)$.
Thus,

$$\begin{aligned} 2n &= \cdots + 1000(10 \cdot t(2d) + u(2d)) + 100(10 \cdot t(2c) + u(2c)) \\ &\qquad + 10(10 \cdot t(2b) + u(2b)) + (10 \cdot t(2a) + u(2a)) \\ &= \cdots + 1000(u(2d) + t(2c)) + 100(u(2c) + t(2b)) + 10(u(2b) + t(2a)) + u(2a) \end{aligned}$$

Since $u(2a), u(2b), u(2c), u(2d) \le 8$ and $t(2a), t(2b), t(2c), t(2d) \le 1$, then each of $u(2d) + t(2c)$ and $u(2c) + t(2b)$ and $u(2b) + t(2a)$ and $u(2a)$ is a single digit, so these are the thousands, hundreds, tens and units digits, respectively, of $2n$.
Thus, the sum of the digits of $2n$ is

$$\begin{aligned} u(2a) + (u(2b) + t(2a)) + (u(2c) + t(2b)) + (u(2d) + t(2c)) + \cdots &= \\ (t(2a) + u(2a)) + (t(2b) + u(2b)) + (t(2c) + u(2c)) + \cdots & \end{aligned}$$

The above argument extends to the remaining digits of n.
In other words, if r is a digit in n, then its contribution to the sum of the digits of $2n$ is the sum of the tens and units digits of $2r$.
Therefore, the digits of n contribute to the sum of the digits of $2n$ as outlined in the table above.

Answer: (D)