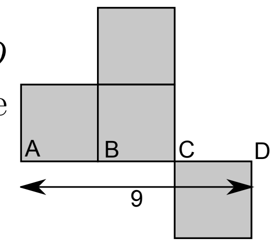
The CENTRE for EDUCATION in MATHEMATICS and COMPUTING
presents the

2012 Canadian Team Mathematics Contest (CTMC)

Individual Problems

1. If $xy + xz = 36$ and $y + z = 9$, what is the value of x?

2. Four congruent squares are drawn as shown. A, B, C and D lie on a straight line and $AD = 9$. What is the area of the shaded region?

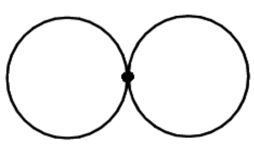


3. The line $x = 2$ intersects the lines $y = -2x + 4$ and $y = \frac{1}{2}x + b$ at points a distance of 1 unit from each other. What are the possible values of b?

4. If $a \Diamond b = \dfrac{b^2}{a}$, for what values of c will $6 \Diamond c = 150$?

5. Two fair dice each have four sides painted blue and two sides painted red. If the two dice are tossed, what is the probability that exactly seven of the blue faces will be visible (that is, not face down on the table)?

6. Two circles each have radius 1 cm. The circles are tangent to each other, and each circle is marked with a dot at the point of tangency.

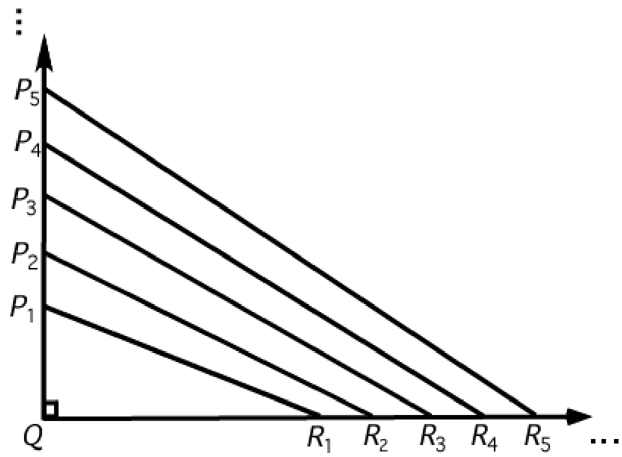


The circles begin turning at the same time, the left circle turning counterclockwise at a constant rate of 2 revolutions per minute, and the right circle turning clockwise at a constant rate of 6 revolutions per minute. The circles turn for one hour. Determine how many times during that hour that the distance between the dots is exactly 4 cm.

7. At homecoming weekend, there is a merry-go-round for children of graduates. The children ride on huge menacing geese that are evenly spaced around a circle. The geese are numbered consecutively from 1 to N where goose 1 follows goose N. A line segment drawn from goose 110 to goose 130 is parallel to a line segment drawn from goose 250 to goose 290. What is the value of N?

CONTINUED ON REVERSE

8. In the diagram, $\triangle QP_1R_1$ is right-angled with $QP_1 = 2$ and $QR_1 = 5$. Lines QP_1 and QR_1 are extended and many more points are labelled at intervals of 1 unit, so that $P_1P_j = j - 1$ for all j and $R_1R_k = k - 1$ for all k.



Determine the value of n so that the area of quadrilateral $P_nP_{n+1}R_{n+1}R_n$ is 2012.

9. Kate is given ten rods each with a different, whole number length. She finds, to her dismay, that she cannot construct a triangle with positive area using any three of the rods. What is the smallest possible length for the longest of the ten rods?

10. An increasing list of two-digit positive integers is formed so that

   - each integer in the list uses only digits from $\{1, 2, 3, 4, 5, 6\}$,
   - each of the integers in the list has the property that its units digit is greater than its tens digit, and
   - each of the digits $\{1, 2, 3, 4, 5, 6\}$ appears in exactly three of the integers in the list.

   How many different lists are possible?