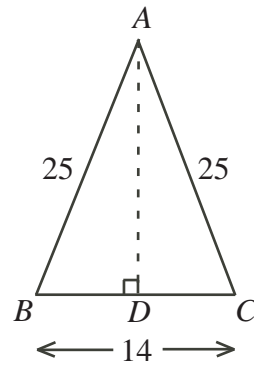


2011 Fryer Contest (Grade 9)
Wednesday, April 13, 2011

1. An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant d , called the common difference. For example, 2, 5, 8, 11, 14 are the first five terms of an arithmetic sequence with a common difference of $d = 3$.
 - (a) Determine the 6th and 7th terms of the sequence given above.
 - (b) What is the 31st term in this sequence?
 - (c) If the last term in this sequence were 110, how many terms would there be in the sequence?
 - (d) If this sequence is continued, does 1321 appear in the sequence? Explain why or why not.

2. In any isosceles triangle ABC with $AB = AC$, the altitude AD bisects the base BC so that $BD = DC$.



- (a)
 - (i) As shown in $\triangle ABC$, $AB = AC = 25$ and $BC = 14$. Determine the length of the altitude AD .
 - (ii) Determine the area of $\triangle ABC$.

- (b) Triangle ABC from part (a) is cut along its altitude from A to D (Figure 1). Each of the two new triangles is then rotated 90° about point D until B meets C directly below D (Figure 2). This process creates the new triangle which is labelled PQR (Figure 3).

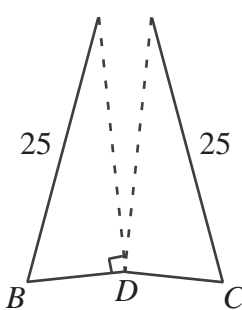


Figure 1

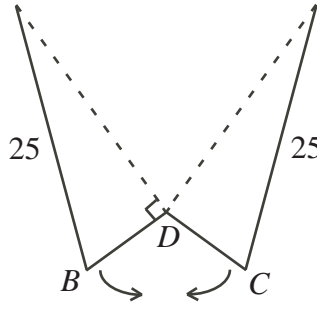


Figure 2

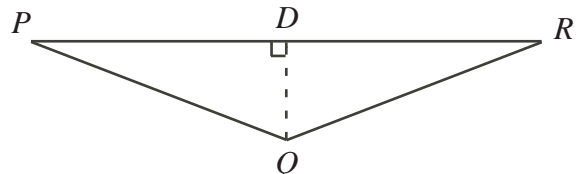
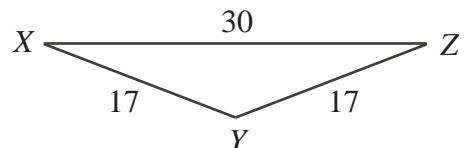


Figure 3

- (i) In $\triangle PQR$, determine the length of the base PR .
 - (ii) Determine the area of $\triangle PQR$.
- (c) There are two different isosceles triangles whose side lengths are integers and whose areas are 120. One of these two triangles, $\triangle XYZ$, is shown. Determine the lengths of the three sides of the second triangle.



3. Begin with any two-digit positive integer and multiply the two digits together. If the resulting product is a two-digit number, then repeat the process. When this process is repeated, all two-digit numbers will eventually become a single digit number. Once a product results in a single digit, the process stops.

For example,

Two-digit number	Step 1	Step 2	Step 3
97	$9 \times 7 = 63$	$6 \times 3 = 18$	$1 \times 8 = 8$
48	$4 \times 8 = 32$	$3 \times 2 = 6$	
50	$5 \times 0 = 0$		

The process stops at 8 after 3 steps.

The process stops at 6 after 2 steps.

The process stops at 0 after 1 step.

- (a) Beginning with the number 68, determine the number of steps required for the process to stop.
- (b) Determine all two-digit numbers for which the process stops at 8 after 2 steps.
- (c) Determine all two-digit numbers for which the process stops at 4.
- (d) Determine a two-digit number for which the process stops after 4 steps.
4. Ian buys a cup of tea every day at Jim Bortons for \$1.72 with money from his coin jar. He starts the year with 365 two-dollar (200¢) coins and no other coins in the jar. Ian makes payment and the cashier provides change according to the following rules:
- Payment is only with money from the coin jar.
 - The amount Ian offers the cashier is at least \$1.72.
 - The amount Ian offers the cashier is as close as possible to the price of the cup of tea.
 - Change is given with the fewest number of coins.
 - Change is placed into the coin jar.
 - Possible coins that may be used have values of 1¢, 5¢, 10¢, 25¢, and 200¢.
- (a) How much money will Ian have in the coin jar after 365 days?
- (b) What is the maximum number of 25¢ coins that Ian could have in the coin jar at any one time?
- (c) How many of each type of coin does Ian have in his coin jar after 277 days?