# The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

# 2011 Fermat Contest

**(Grade 11)**

**Thursday, February 24, 2011**

*Solutions*

1. Evaluating, $\dfrac{2+3\times 6}{23+6} = \dfrac{2+18}{29} = \dfrac{20}{29}$.

   Answer: (D)

2. If $y = 77$, then $\dfrac{7y+77}{77} = \dfrac{7y}{77} + \dfrac{77}{77} = \dfrac{7(77)}{77} + 1 = 7 + 1 = 8$.

   Answer: (A)

3. Since the area of the rectangle is 192 and its length is 24, then its width is $192 \div 24 = 8$.
   Therefore, its perimeter is $2 \times 24 + 2 \times 8 = 64$.

   Answer: (A)

4. Since $\sqrt{n+9} = 25$, then $n + 9 = 25^2 = 625$.
   Thus, $n = 625 - 9 = 616$.

   Answer: (D)

5. Since $\triangle PRS$ is equilateral, then all three of its angles equal $60^\circ$.
   In particular, $\angle RSP = 60^\circ$.
   Since $QS = QT$, then $\triangle QST$ is isosceles and so $\angle TSQ = \angle STQ = 40^\circ$.
   Since $RST$ is a straight line segment, then $\angle RSP + \angle PSQ + \angle TSQ = 180^\circ$.
   Therefore, $60^\circ + x^\circ + 40^\circ = 180^\circ$ or $x = 180 - 60 - 40 = 80$.

   Answer: (C)

6. If the sum of three consecutive integers is 27, then the numbers must be 8, 9 and 10. (We could see this algebraically by calling the integers $x$, $x+1$ and $x+2$ and solving the equation $x + (x+1) + (x+2) = 27$.)
   Their product is $8 \times 9 \times 10 = 720$.

   Answer: (C)

7. The number halfway between two numbers is their average.
   Therefore, the number halfway between $\frac{1}{10}$ and $\frac{1}{12}$ is $\frac{1}{2}(\frac{1}{10} + \frac{1}{12}) = \frac{1}{2}(\frac{12}{120} + \frac{10}{120}) = \frac{1}{2}(\frac{22}{120}) = \frac{11}{120}$.

   Answer: (D)

8. Since the angle in the sector representing cookies is $90^\circ$, then this sector represents $\frac{1}{4}$ of the total circle.
   Therefore, 25% of the students chose cookies as their favourite food.
   Thus, the percentage of students who chose sandwiches was $100\% - 30\% - 25\% - 35\% = 10\%$.
   Since there are 200 students in total, then $200 \times \frac{10}{100} = 20$ students said that their favourite food was sandwiches.

   Answer: (B)

9. The set $S$ contains 25 multiples of 2 (that is, even numbers).
   When these are removed, the set $S$ is left with only the odd integers from 1 to 49.
   At this point, there are $50 - 25 = 25$ integers in $S$.
   We still need to remove the multiples of 3 from $S$.
   Since $S$ only contains odd integers at this point, then we must remove the odd multiples of 3 between 1 and 49.
   These are 3, 9, 15, 21, 27, 33, 39, 45, of which there are 8.
   Therefore, the number of integers remaining in the set $S$ is $25 - 8 = 17$.

   Answer: (D)

10. *Solution 1*
Since $PQRS$ is a square and $QR = 2 + 9 = 11$, then $PQ = QR = SR = PS = 11$.
The height of the shaded rectangle equals the height of the top left rectangle minus the height of the top right rectangle, or $6 - 2 = 4$.
The width of the shaded rectangle equals the width of the top right rectangle minus the width of the bottom right rectangle.
Since $SR = 11$, then the width of the bottom right rectangle is $11 - 10 = 1$.
Therefore, the width of the shaded rectangle is $8 - 1 = 7$.
Thus, the area of the shaded rectangle is $4 \times 7 = 28$.

*Solution 2*
Since $PQRS$ is a square and $QR = 2 + 9 = 11$, then $PQ = QR = SR = PS = 11$.
Since the side length of the square is 11, then its area is $11^2 = 121$.
Since $PQ = 11$, then the width of the top left rectangle is $11 - 8 = 3$, and so its area is $3 \times 6 = 18$.
Since $PS = 11$, then the height of the bottom left rectangle is $11 - 6 = 5$, and so its area is $5 \times 10 = 50$.
Since $SR = 11$, then the width of the bottom right rectangle is $11 - 10 = 1$, and so its area is $1 \times 9 = 9$.
The area of the top right rectangle is $8 \times 2 = 16$.
Thus, the area of the shaded rectangle equals the area of square $PQRS$ minus the combined areas of the four unshaded rectangles, or $121 - 18 - 50 - 9 - 16 = 28$.

ANSWER: (B)

11. It is possible that after buying 7 gumballs, Wally has received 2 red, 2 blue, 1 white, and 2 green gumballs.
This is the largest number of each colour that he could receive without having three gumballs of any one colour.
If Wally buys another gumball, he will receive a blue or a green or a red gumball.
In each of these cases, he will have at least 3 gumballs of one colour.
In summary, if Wally buys 7 gumballs, he is not guaranteed to have 3 of any one colour; if Wally buys 8 gumballs, he is guaranteed to have 3 of at least one colour.
Therefore, the least number that he must buy to guarantee receiving 3 of the same colour is 8.

ANSWER: (E)

12. *Solution 1*
A parabola is symmetric about its axis of symmetry.
Since the $x$-intercepts of the given parabola are $x = -1$ and $x = 4$, then the axis of symmetry of the parabola is $x = \frac{-1+4}{2} = \frac{3}{2}$.
Since the point $(3, w)$ is $\frac{3}{2}$ units to the right of the axis of symmetry, then its $y$-coordinate (namely $w$) equals the $y$-coordinate of the point $\frac{3}{2}$ units to the left of the axis of symmetry, which is the point with $x = 0$.
When $x = 0$, we know that $y = 8$.
Therefore, $w = 8$.
(We could also note that $x = 3$ is 1 unit to the left of the rightmost $x$-intercept so its $y$-coordinate is equal to that of the point 1 unit to the right of the leftmost $x$-intercept.)

*Solution 2*

Since the parabola has $x$-intercepts of $-1$ and 4, then its equation is of the form $y = a(x+1)(x-4)$ for some value of $a$.
Since the point $(0, 8)$ lies on the parabola, then $8 = a(1)(-4)$ or $a = -2$.
Therefore, the parabola has equation $y = -2(x+1)(x-4)$.
Since the point $(3, w)$ lies on the parabola, then $w = -2(4)(-1) = 8$.

ANSWER: (E)

13. Since Xavier, Yolanda and Zixuan have \$50 in total, and the ratio of the amount that Xavier has to the amount that the other two have is 3 : 2, then Xavier has $\frac{3}{5}$ of the total, or $\frac{3}{5} \times \$50 = \$30$.
Therefore, Yolanda and Zixuan together have $\$50 - \$30 = \$20$.
We know that Yolanda has \$4 more than Zixuan, so we must break \$20 into two parts, one of which is \$4 larger than the other.
If Yolanda has \$12 and Zixuan has \$8, this satisfies the requirements.
Therefore, Zixuan has \$8.

ANSWER: (B)

14. The average of two multiples of 4 must be even, since we can write these multiples of 4 as $4m$ and $4n$ for some integers $m$ and $n$, which means that their average is $\frac{1}{2}(4m + 4n)$ which equals $2m + 2n$ or $2(m + n)$, which is a multiple of 2, and so is even.
Each of the other four choices may be an odd integer in some cases. Here is an example for each:

(A) The average of 2 and 4 is 3, which is not even
(B) The average of 3 and 7 is 5, which is not even
(C) The average of 1 and 9 is 5, which is not even
(E) The average of 2, 3 and 4 is 3, which is not even

Therefore, the correct answer is (D).

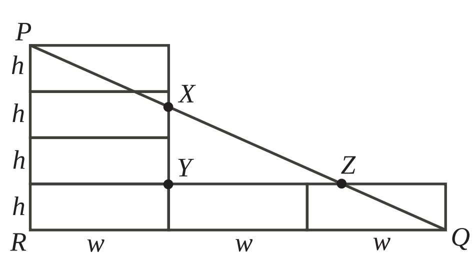
ANSWER: (D)

15. Since $m$ and $n$ are consecutive positive integers with $n^2 - m^2 > 20$, then $n$ is greater than $m$.
Therefore, we can write $n = m + 1$.
Since $n^2 - m^2 > 20$, then $(m+1)^2 - m^2 > 20$ or $m^2 + 2m + 1 - m^2 > 20$ or $2m > 19$ or $m > \frac{19}{2}$.
Since $m$ is a positive integer, then $m \geq 10$.
Thus, we want to find the minimum value of $n^2 + m^2 = (m+1)^2 + m^2 = 2m^2 + 2m + 1$ when $m \geq 10$.
This minimum will occur when $m = 10$ (since $2m^2 + 2m + 1$ increases with $m$ when $m$ is a positive integer).
Therefore, the minimum possible value is $2(10^2) + 2(10) + 1 = 221$.

ANSWER: (E)

16. *Solution 1*

We label the bottom left corner as $R$ and label various side lengths as $h$ and $w$:



Since the diagram is made up of rectangles, then $XY$ is parallel to $PR$, which tells us that $\angle YXZ = \angle RPQ$. Also, $YZ$ is parallel to $RQ$, which tells us that $\angle XZY = \angle PQR$.
Therefore, $\triangle PRQ$ is similar to $\triangle XYZ$.
Thus, $\dfrac{RQ}{PR} = \dfrac{YZ}{XY}$.
But $YZ = 2XY$, $RQ = 3w$ and $PR = 4h$.
This tells us that $\dfrac{3w}{4h} = \dfrac{2XY}{XY}$ or $\dfrac{3}{4} \cdot \dfrac{w}{h} = 2$ or $\dfrac{h}{w} = \dfrac{3}{8}$.

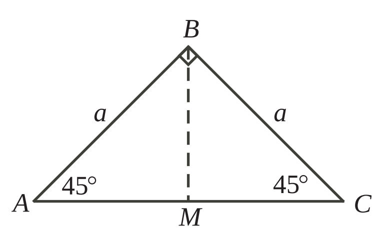
*Solution 2*
As in Solution 1, we label the bottom left corner as $R$ and label various side lengths as $h$ and $w$.
Since $YZ = 2XY$, then the slope of line segment $XZ$ is $\dfrac{XY}{YZ} = \dfrac{XY}{2XY} = \dfrac{1}{2}$.
Since $PR = 4h$ and $RQ = 3w$, then the slope of line segment $PQ$ is $\dfrac{PR}{RQ} = \dfrac{4h}{3w}$.
Since line segment $XZ$ is a portion of line segment $PQ$, then the slopes of these two line segments are equal, so $\dfrac{4h}{3w} = \dfrac{1}{2}$ and so $\dfrac{h}{w} = \dfrac{1}{2} \cdot \dfrac{3}{4} = \dfrac{3}{8}$.

Answer: (C)

17. Since $3^{2x} = 64$ and $3^{2x} = (3^x)^2$, then $(3^x)^2 = 64$ and so $3^x = \pm 8$.
Since $3^x > 0$, then $3^x = 8$.
Thus, $3^{-x} = \dfrac{1}{3^x} = \dfrac{1}{8}$.

Answer: (E)

18. We label the stages in this process as Stage 0 (a square), Stage 1 (2 triangles), Stage 2 (4 triangles), Stage 3 (8 triangles), and Stage 4 (16 triangles).
We want to determine the length of the longest edge of one of the 16 triangles in Stage 4.
At Stage 1, we have two right-angled isosceles triangles with legs of length 4.
Consider a general right-angled isosceles triangle $ABC$ with legs $AB$ and $BC$ of length $a$.
Since this is a 45°-45°-90° triangle, its hypotenuse $AC$ has length $\sqrt{2}$a.
We split the triangle into two equal pieces by bisecting the right-angle at $B$:




Since $\triangle ABC$ is isosceles, then this bisecting line is both an altitude and a median. In other words, it is perpendicular to $AC$ at $M$ and $M$ is the midpoint of $AC$.
Therefore, the two triangular pieces $\triangle AMB$ and $\triangle CMB$ are identical 45°-45°-90° triangles.
The longest edges of these triangles ($AB$ and $CB$) are the legs of the original triangle, and so have length $a$.
Since the longest edge of the original triangle was $\sqrt{2}a$, then the longest edge has been reduced by a factor of $\sqrt{2}$.
Since we have shown that this is the case for an arbitrary isosceles right-angled triangle, we can then apply this property to our problem.
In Stage 1, the longest edge has length $4\sqrt{2}$.

Since the longest edge in Stage 1 has length $4\sqrt{2}$, then the longest edge in Stage 2 has length $\frac{4\sqrt{2}}{\sqrt{2}} = 4$.
Since the longest edge in Stage 2 has length 4, then the longest edge in Stage 3 has length $\frac{4}{\sqrt{2}} = \frac{2\sqrt{2}\sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$.
Since the longest edge in Stage 3 has length $2\sqrt{2}$, then the longest edge in Stage 4 has length $\frac{2\sqrt{2}}{\sqrt{2}} = 2$.

ANSWER: (B)

19. Suppose that the radius of the larger circle is $r$.
Join $O$ to $P$. Then $OP = OS = r$.
Since $Q$ is the midpoint of $PR$, and $PR = 12$, then $PQ = \frac{1}{2}PR = 6$.
Since $OS = r$ and $QS = 4$, then $OQ = OS - QS = r - 4$.
Since $\triangle OPQ$ is right-angled at $Q$, then by the Pythagorean Theorem,

$$\begin{aligned} OQ^2 + PQ^2 &= OP^2 \\ (r-4)^2 + 6^2 &= r^2 \\ r^2 - 8r + 16 + 36 &= r^2 \\ 52 &= 8r \\ r &= \tfrac{52}{8} \end{aligned}$$

Therefore, the radius of the larger circle is $\frac{52}{8}$, or 6.5.

ANSWER: (C)

20. Since $b = ar$, $c = ar^2$, and the product of $a$, $b$ and $c$ is 46656, then $a(ar)(ar^2) = 46656$ or $a^3r^3 = 46656$ or $(ar)^3 = 46656$ or $ar = \sqrt[3]{46656} = 36$.
Therefore, $b = ar = 36$.
Since the sum of $a$, $b$ and $c$ is 114, then $a + c = 114 - b = 114 - 36 = 78$.

ANSWER: (A)

21. In the given pattern, the $r$th row contains $r$ integers.
Therefore, after $n$ rows, the total number of integers appearing in the pattern is

$$1 + 2 + 3 + \cdots + (n-2) + (n-1) + n$$

This expression is always equal to $\frac{1}{2}n(n+1)$.
(If you have never seen this formula before, try to prove it!)
Putting this another way, the largest number in the $n$th row is $\frac{1}{2}n(n+1)$.
To determine which row the number 400 is in, we want to determine the smallest value of $n$ for which $\frac{1}{2}n(n+1) \geq 400$ or $n(n+1) \geq 800$.
If $n = 27$, then $n(n+1) = 756$.
If $n = 28$, then $n(n+1) = 812$.
Therefore, 400 appears in the 28th row. Also, the largest integer in the 28th row is 406 and the largest integer in the 27th row is 378.
Thus, we want to determine the sum of the integers from 379 (the first integer in the 28th row) to 406, inclusive.
We can do this by calculating the sum of the integers from 1 to 406 and subtracting the sum of the integers from 1 to 378.
Since the sum of the integers from 1 to $m$ equals $\frac{1}{2}m(m+1)$, then the sum of the integers from 379 to 406 is equal to $\frac{1}{2}(406)(407) - \frac{1}{2}(378)(379) = 10990$.

ANSWER: (A)

22. Since $\dfrac{p+q^{-1}}{p^{-1}+q} = 17$, then $\dfrac{p+\dfrac{1}{q}}{\dfrac{1}{p}+q} = 17$ or $\dfrac{\dfrac{pq+1}{q}}{\dfrac{1+pq}{p}} = 17$ or $\dfrac{p(pq+1)}{q(pq+1)} = 17$.

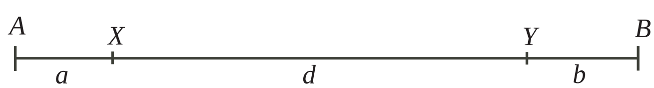
Since $p$ and $q$ are positive integers, then $pq+1>0$, so we can divide out the common factor in the numerator and denominator to obtain $\dfrac{p}{q} = 17$ or $p = 17q$.
Since $p$ and $q$ are positive integers, then $q \geq 1$.
Since $p+q \leq 100$, then $17q+q \leq 100$ or $18q \leq 100$ or $q \leq \frac{100}{18} = 5\frac{5}{9}$.
Since $q$ is a positive integer, then $q \leq 5$.
Therefore, the combined restriction is $1 \leq q \leq 5$, and so there are five pairs.
(We can check that these pairs are $(p,q) = (17,1),(34,2),(51,3),(68,4),(85,5)$.)

ANSWER: (E)

23. First, we note that the three people are interchangeable in this problem, so it does not matter who rides and who walks at any given moment. We abbreviate the three people as D, M and P.
We call their starting point $A$ and their ending point $B$.
Here is a strategy where all three people are moving at all times and all three arrive at $B$ at the same time:

D and M get on the motorcycle while P walks.
D and M ride the motorcycle to a point $Y$ before $B$.
D drops off M and rides back while P and M walk toward $B$.
D meets P at point $X$.
D picks up P and they drive back to $B$ meeting M at $B$.
Point $Y$ is chosen so that D, M and P arrive at $B$ at the same time.

Suppose that the distance from $A$ to $X$ is $a$ km, from $X$ to $Y$ is $d$ km, and the distance from $Y$ to $B$ is $b$ km.




In the time that it takes P to walk from $A$ to $X$ at 6 km/h, D rides from $A$ to $Y$ and back to $X$ at 90 km/h.
The distance from $A$ to $X$ is $a$ km.
The distance from $A$ to $Y$ and back to $X$ is $a+d+d = a+2d$ km.
Since the time taken by P and by D is equal, then $\dfrac{a}{6} = \dfrac{a+2d}{90}$ or $15a = a+2d$ or $7a = d$.
In the time that it takes M to walk from $Y$ to $B$ at 6 km/h, D rides from $Y$ to $X$ and back to $B$ at 90 km/h.
The distance from $Y$ to $B$ is $b$ km, and the distance from $Y$ to $X$ and back to $B$ is $d+d+b = b+2d$ km.
Since the time taken by M and by D is equal, then $\dfrac{b}{6} = \dfrac{b+2d}{90}$ or $15b = b+2d$ or $7b = d$.
Therefore, $d = 7a = 7b$, and so we can write $d = 7a$ and $b = a$.
Thus, the total distance from $A$ to $B$ is $a+d+b = a+7a+a = 9a$ km.
However, we know that this total distance is 135 km, so $9a = 135$ or $a = 15$.
Finally, D rides from $A$ to $Y$ to $X$ to $B$, a total distance of $(a+7a)+7a+(7a+a) = 23a$ km.

Since $a = 15$ km and D rides at 90 km/h, then the total time taken for this strategy is $\frac{23\times 15}{90} = \frac{23}{6} \approx 3.83$ h.
Since we have a strategy that takes 3.83 h, then the smallest possible time is no more than 3.83 h. Can you explain why this is actually the smallest possible time?

If we didn't think of this strategy, another strategy that we might try would be:

D and M get on the motorcycle while P walks.
D and M ride the motorcycle to $B$.
D drops off M at $B$ and rides back to meet P, who is still walking.
D picks up P and they drive back to $B$. (M rests at $B$.)

This strategy actually takes 4.125 h, which is longer than the strategy shown above, since M is actually sitting still for some of the time.

Answer: (A)

24. The six possible sums are $w+x$, $w+y$, $w+z$, $x+y$, $x+z$, and $y+z$.
Since $x<y$, then $w+x<w+y$.
Since $w<x$, then $w+y<x+y$.
Since $y<z$, then $x+y<x+z$.
Since $x<y$, then $x+z<y+z$.
Therefore, we have $w+x<w+y<x+y<x+z<y+z$.
This list includes all of the sums except $w+z$.
Since $y<z$ and $w<x$, then $w+y<w+z<x+z$, but we cannot say for sure whether $x+y$ or $w+z$ is larger.
Thus, we know that $w+x$ is always the smallest sum and that $w+y$ is always the second smallest sum. Also, we know that the third and fourth smallest sums are $w+z$ and $x+y$ in some order.
We can conclude that $w+x=1$ and $w+y=2$, and $w+z$ and $x+y$ equal 3 and 4 in some order.
From the first and second equations, $(w+y)-(w+x)=2-1$ or $y-x=1$.

Case 1: $w+z=3$ and $x+y=4$
Since $y-x=1$ and $x+y=4$, we add these to obtain $2y=5$ or $y=\frac{5}{2}$.
Since $w+y=2$, then $w=2-y=2-\frac{5}{2}=-\frac{1}{2}$.
Since $w+z=3$, then $z=3-w=3-(-\frac{1}{2})=\frac{7}{2}$.
Since $x+y=4$, then $x=4-y=4-\frac{5}{2}=\frac{3}{2}$.
Therefore, we have $w=-\frac{1}{2}$, $x=\frac{3}{2}$, $y=\frac{5}{2}$, and $z=\frac{7}{2}$.
We can check that the six sums are $1,2,3,4,5,6$, which are all different as required.

Case 2: $w+z=4$ and $x+y=3$
Since $y-x=1$ and $x+y=3$, we add these to obtain $2y=4$ or $y=2$.
Since $w+y=2$, then $w=2-y=2-2=0$.
Since $w+z=4$, then $z=4-w=4-0=4$.
Since $x+y=3$, then $x=3-y=3-2=1$.
Therefore, we have $w=0$, $x=1$, $y=2$, and $z=4$.
We can check that the six sums are $1,2,3,4,5,6$, which are all different as required.

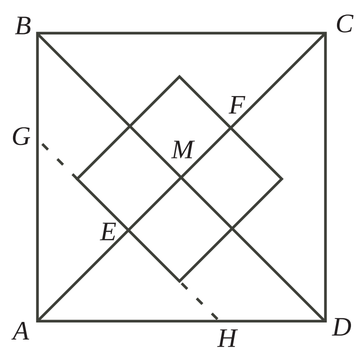
Therefore, the two possible values of $z$ are 4 and $\frac{7}{2}$.
The sum of these values is $4+\frac{7}{2}=\frac{15}{2}$.

Answer: (D)

25. The smallest possible height of the pyramid will occur when the four side faces are just touching the circumference of the end faces of the cylinder. To see this, consider starting with the top vertex of the pyramid much higher than in its position with minimum height. In this higher position, none of the lateral faces of the pyramid touch the cylinder. We gradually lower this top vertex towards the centre of the square base. Eventually, each of the lateral faces of the pyramid will touch the "rim" of one of the circular ends of the cylinder. We cannot lower the top vertex any further since otherwise part of the cylinder would be outside of the pyramid. This is our minimum height position. We calculate the height of this pyramid.

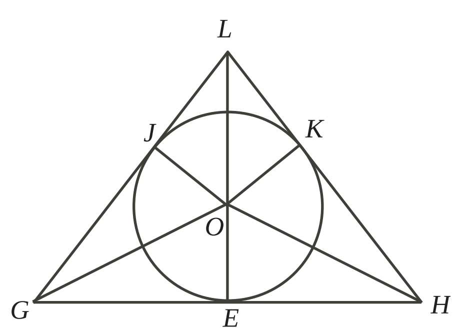
We label the square base of the pyramid as $ABCD$, and the top vertex of the pyramid as $T$. Join $AC$ and $BD$, the diagonals of the base. Label their point of intersection, which is also the centre of the base, as $M$.
Since the square base has side length 20, then $AC = BD = 20\sqrt{2}$.
Since the diagonals bisect each other, then $AM = BM = CM = DM = 10\sqrt{2}$.
Note that $T$ lies directly above $M$.
Let $t$ be the height of the pyramid; that is, let $t = TM$. We want to calculate $t$.

Suppose that the central axis of the cylinder lies above $AC$.
Since the midpoint of the central axis lies directly above $M$, then the central axis extends a distance of 5 to either side of $M$.
Label the points in contact with $AC$ at the two ends of the cylinder as $E$ and $F$. Since $EM = FM = 5$, then each end of the cylinder lies a distance of $10\sqrt{2} - 5$ from the corner of the base, as measured along the diagonal (that is, $AE = 10\sqrt{2} - 5$).




From above, the cylinder's "footprint" on the base of the pyramid is actually a square, since its diameter becomes its width.

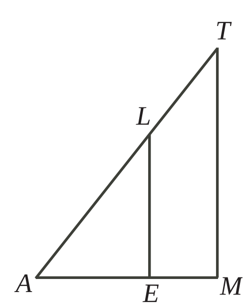
Consider a vertical cross section of the pyramid and cylinder through the end of the cylinder closest to $A$.



Let $L$ be the point where the cross-section intersects $AT$ and $G$ and $H$ be the points where the cross-section intersects $AB$ and $AD$, respectively. These points $G$ and $H$ are the same points shown in the first diagram.
Since $\angle BAM = 45^\circ$ and the cylinder's face is perpendicular to the diagonal of the square base, then $\triangle GEA$ is isosceles and right-angled (as is $\triangle HEA$) so $GE = HE = AE = 10\sqrt{2} - 5$.
Let $O$ be the centre of the end face of the cylinder.
Note that $GL$ and $HL$ are lines that lie along the faces $ABT$ and $ADT$ of the pyramid.
Since the faces $ABT$ and $ADT$ of the pyramid just touch the cylinder's rim, then $GL$ and $HL$ are tangent to the circular face, say at $J$ and $K$, respectively.
Join $O$ to points $G$, $H$ and $L$.
Also, join $O$ to points $J$, $K$ and $E$. Each of these segments is a radius of the circular face, so each has length 5.
Since the circle is tangent to faces of the pyramid (including the bottom face) at these points, then each of these segments is perpendicular to the corresponding side of $\triangle GHL$.
Our goal will be to calculate the length of $LE$.
Since $GE$ and $GJ$ are tangents to the circle from a common point, then $GJ = GE = 10\sqrt{2} - 5$.
Let $LE = h$. Then $LO = h - 5$. Also, let $LJ = x$.
Note that $\triangle LJO$ is similar to $\triangle LEG$, since they have a common angle at $L$ and each is right-angled.
Since these triangles are similar, then $\dfrac{LJ}{JO} = \dfrac{LE}{EG}$.
Therefore, $\dfrac{x}{5} = \dfrac{h}{10\sqrt{2} - 5}$ or $x = \dfrac{5h}{10\sqrt{2} - 5} = \dfrac{h}{2\sqrt{2} - 1}$.
Also, from the similarity, $\dfrac{LG}{GE} = \dfrac{LO}{OJ}$.
Therefore, $\dfrac{x + (10\sqrt{2} - 5)}{10\sqrt{2} - 5} = \dfrac{h - 5}{5}$ or $x + (10\sqrt{2} - 5) = (2\sqrt{2} - 1)(h - 5)$.
Substituting $x = \dfrac{h}{2\sqrt{2} - 1}$, we obtain

$$
\begin{aligned}
\frac{h}{2\sqrt{2} - 1} + (10\sqrt{2} - 5) &= (2\sqrt{2} - 1)(h - 5) \\
h + 5(2\sqrt{2} - 1)^2 &= (2\sqrt{2} - 1)^2(h - 5) \\
h + 5(2\sqrt{2} - 1)^2 &= (9 - 4\sqrt{2})h - 5(2\sqrt{2} - 1)^2 \\
10(2\sqrt{2} - 1)^2 &= (8 - 4\sqrt{2})h \\
h &= \frac{10(2\sqrt{2} - 1)^2}{8 - 4\sqrt{2}}
\end{aligned}
$$

Finally, to calculate $t$, we extract $\triangle AMT$.



Note that $E$ lies on $AM$ and $L$ lies on $AT$.
Also, $TM$ is perpendicular to $AM$ and $LE$ is perpendicular to $AE$, which means that $\triangle AEL$ is similar to $\triangle AMT$.
Therefore, $\dfrac{TM}{AM} = \dfrac{LE}{AE}$, or $\dfrac{t}{10\sqrt{2}} = \dfrac{h}{10\sqrt{2}-5}$, and so

$$t = \frac{10\sqrt{2}}{5(2\sqrt{2}-1)} \cdot \frac{10(2\sqrt{2}-1)^2}{8-4\sqrt{2}} = \frac{20\sqrt{2}(2\sqrt{2}-1)}{8-4\sqrt{2}} = \frac{5\sqrt{2}(2\sqrt{2}-1)}{2-\sqrt{2}} \approx 22.07$$

Of the given answers, the smallest possible height is closest to 22.1.

Answer: (B)