The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Cayley Contest

(Grade 10)

Thursday, February 24, 2011

Time: 60 minutes
Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $(5+2)+(8+6)+(4+7)+(3+2)$ is

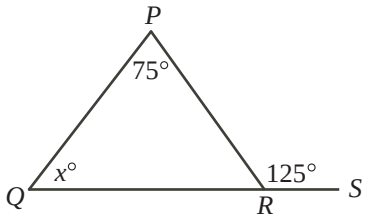
   (A) 35 **(B)** 37 **(C)** 40 **(D)** 45 **(E)** 47

2. If $(-1)(2)(x)(4) = 24$, then x equals

   (A) 4 **(B)** -3 **(C)** -1 **(D)** 2 **(E)** -4

3. In the diagram, R lies on line segment QS. What is the value of x?

   (A) 50 **(B)** 55 **(C)** 75
   (D) 100 **(E)** 105



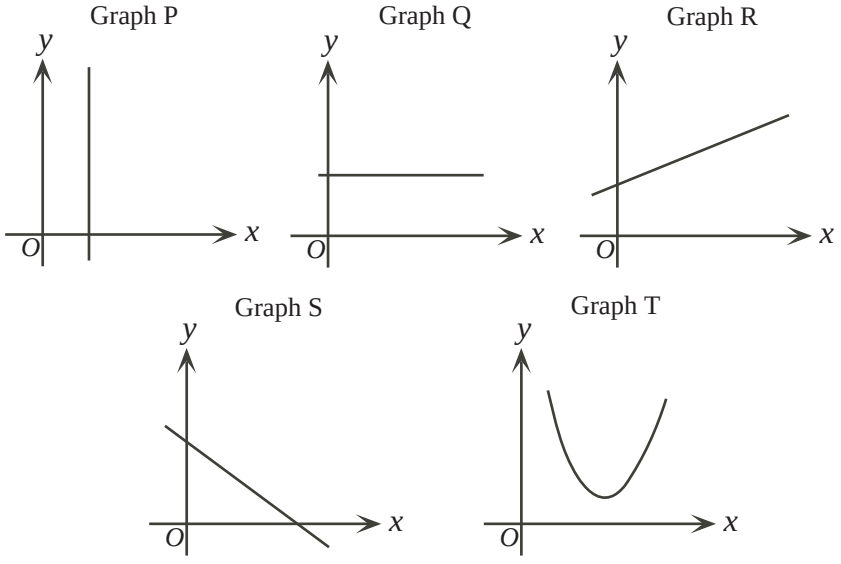
4. When a number is tripled, then decreased by 5, the result is 16. What is the original number?

   (A) 3 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

5. The expression $\sqrt{13+\sqrt{7+\sqrt{4}}}$ is equal to

   (A) 7 **(B)** 8 **(C)** 6 **(D)** 4 **(E)** 5

6. Which of the five graphs is linear with a slope of 0?



   (A) Graph P **(B)** Graph Q **(C)** Graph R **(D)** Graph S **(E)** Graph T

7. After a fair die with faces numbered 1 to 6 is rolled, the number on the top face is x. Which of the following is most likely?

(A) x is greater than 2 **(B)** x equals 4 or 5 **(C)** x is even
(D) x is less than 3 **(E)** x equals 3

8. If 2.4×10^8 is doubled, then the result is equal to

(A) 2.4×20^8 **(B)** 4.8×20^8 **(C)** 4.8×10^8 **(D)** 2.4×10^{16} **(E)** 4.8×10^{16}

9. A proposed new \$5 coin is called the "foonie". The foonie's two faces are identical and each has area 5 cm^2. The thickness of the foonie is 0.5 cm. How many foonies are in a stack that has a volume of 50 cm^3?

(A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 40

10. The Athenas are playing a 44 game season. Each game results in a win or a loss, and cannot end in a tie. So far, they have 20 wins and 15 losses. In order to make the playoffs, they must win at least 60% of all of their games. What is the smallest number of their remaining games that they must win to make the playoffs?

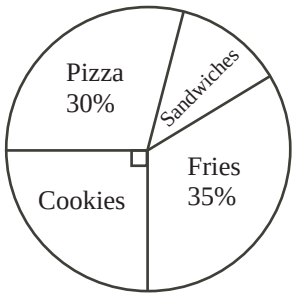
(A) 8 **(B)** 9 **(C)** 5 **(D)** 6 **(E)** 7

Part B: Each correct answer is worth 6.

11. The operation "∇" is defined by $(a, b)\nabla(c, d) = ac + bd$.
For example $(1, 2)\nabla(3, 4) = (1)(3) + (2)(4) = 11$.
The value of $(3, 1)\nabla(4, 2)$ is

(A) 10 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 24

12. The circle graph shown illustrates the results of a survey taken by the Cayley H.S. Student Council to determine the favourite cafeteria food. How many of the 200 students surveyed said that their favourite food was sandwiches?



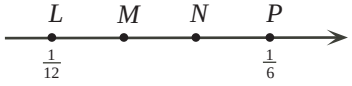
(A) 10 **(B)** 20 **(C)** 35
(D) 50 **(E)** 70

13. In the subtraction shown, K, L, M, and N are digits. What is the value of $K + L + M + N$?

$$\begin{array}{rcccc} & 5 & K & 3 & L \\ - & M & 4 & N & 1 \\ \hline & 4 & 4 & 5 & 1 \end{array}$$

(A) 20 **(B)** 19 **(C)** 16
(D) 13 **(E)** 9

14. On the number line, points M and N divide LP into three equal parts. What is the value at M?



(A) $\frac{1}{7}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{1}{9}$
(D) $\frac{1}{10}$ **(E)** $\frac{1}{11}$

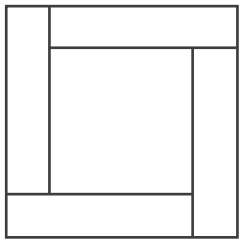
15. The points $Q(1,-1)$, $R(-1,0)$ and $S(0,1)$ are three vertices of a parallelogram. The coordinates of the fourth vertex of the parallelogram could be

(A) $(-2,2)$ **(B)** $(0,-1)$ **(C)** $(0,0)$ **(D)** $(\frac{3}{2},\frac{1}{2})$ **(E)** $(-1,1)$

16. A gumball machine that randomly dispenses one gumball at a time contains 13 red, 5 blue, 1 white, and 9 green gumballs. What is the least number of gumballs that Wally must buy to *guarantee* that he receives 3 gumballs of the same colour?

(A) 6 **(B)** 9 **(C)** 4 **(D)** 7 **(E)** 8

17. Four congruent rectangles and a square are assembled without overlapping to form a large square, as shown. Each of the rectangles has a perimeter of 40 cm. The total area of the large square is

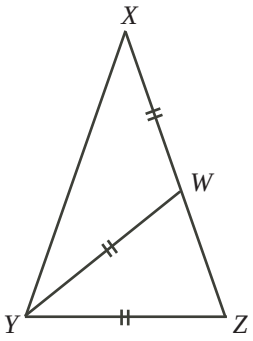


(A) 160 cm^2 **(B)** 200 cm^2 **(C)** 400 cm^2
(D) 800 cm^2 **(E)** 1600 cm^2

18. When 100 is divided by 12, the remainder is 4.
When 100 is divided by a positive integer x, the remainder is 10.
When 1000 is divided by x, the remainder is

(A) 10 **(B)** 100 **(C)** 0 **(D)** 1 **(E)** 90

19. In the diagram, $\triangle XYZ$ is isosceles with $XY = XZ$. Also, point W is on XZ so that $XW = WY = YZ$. The measure of $\angle XYW$ is



(A) 18° **(B)** 30° **(C)** 45°
(D) 36° **(E)** 60°

20. For how many positive integers n, with $n \leq 100$, is n^3+5n^2 the square of an integer?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Part C: Each correct answer is worth 8.

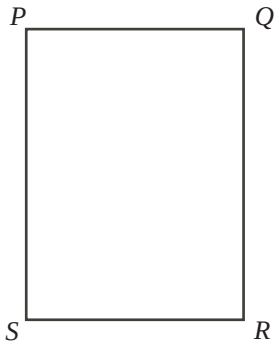
21. Suppose that x and y are positive numbers with

$$\begin{aligned} xy &= \tfrac{1}{9} \\ x(y+1) &= \tfrac{7}{9} \\ y(x+1) &= \tfrac{5}{18} \end{aligned}$$

What is the value of $(x+1)(y+1)$?

(A) $\frac{11}{6}$ **(B)** $\frac{8}{9}$ **(C)** $\frac{16}{9}$ **(D)** $\frac{10}{9}$ **(E)** $\frac{35}{18}$

22. The top section of an 8 cm by 6 cm rectangular sheet of paper is folded along a straight line so that when the top section lies flat on the bottom section, corner P lies on top of corner R. The length of the crease, in cm, is



(A) 6.25 **(B)** 7 **(C)** 7.5
(D) 7.4 **(E)** 10

23. A *Fano table* is a table with three columns where

- each entry is an integer taken from the list $1, 2, 3, \ldots, n$, and
- each row contains three different integers, and
- for each possible pair of distinct integers from the list $1, 2, 3, \ldots, n$, there is exactly one row that contains both of these integers.

1	2	4
2	3	5
3	4	6
4	5	7
5	6	1
6	7	2
7	1	3

The number of rows in the table will depend on the value of n. For example, the table shown is a Fano table with $n = 7$. (Notice that 2 and 6 appear in the same row only once, as does every other possible pair of the numbers $1, 2, 3, 4, 5, 6, 7$.) For how many values of n with $3 \leq n \leq 12$ can a Fano table be created?

(A) 2 **(B)** 3 **(C)** 5
(D) 6 **(E)** 7

24. Dolly, Molly and Polly each can walk at 6 km/h. Their one motorcycle, which travels at 90 km/h, can accommodate at most two of them at once (and cannot drive by itself!). Let t hours be the time taken for all three of them to reach a point 135 km away. Ignoring the time required to start, stop or change directions, what is true about the smallest possible value of t?

(A) $t < 3.9$ **(B)** $3.9 \leq t < 4.1$ **(C)** $4.1 \leq t < 4.3$
(D) $4.3 \leq t < 4.5$ **(E)** $t \geq 4.5$

25. Two numbers a and b with $0 \leq a \leq 1$ and $0 \leq b \leq 1$ are chosen at random. The number c is defined by $c = 2a + 2b$. The numbers a, b and c are each rounded to the nearest integer to give A, B and C, respectively. (For example, if $a = 0.432$ and $b = 0.5$, then $c = 1.864$, and so $A = 0$, $B = 1$, and $C = 2$.) What is the probability that $2A + 2B = C$?

(A) $\frac{15}{32}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{3}{4}$

2011
Cayley
Contest
(English)

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