The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
www.cemc.uwaterloo.ca

Canadian Intermediate Mathematics Contest

Sample Contest

Time: 2 hours

Calculators are permitted, provided they are non-programmable and without graphic displays.

Do not open this booklet until instructed to do so.
There are two parts to this paper.

PART A

1. This part consists of 6 questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of 3 questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, it will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

NOTES:

At the completion of the contest, insert the information sheet inside the answer booklet.

The names of some top-scoring students will be published on the CEMC website, http://www.cemc.uwaterloo.ca.

Canadian Intermediate Mathematics Contest

NOTE:	1.	Please read the instructions on the front cover of this booklet.
	2.	Write solutions in the answer booklet provided.
	3.	It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2+\sqrt{7}$, etc., rather than as $12.566\ldots$ or $4.646\ldots$.
	4.	**Calculators are permitted**, provided they are non-programmable and without graphic displays.
	5.	Diagrams are not drawn to scale. They are intended as aids only.

Note: The problems for this sample contest have been taken from past CEMC contests to demonstrate the level of difficulty that the 2011 contest will have.

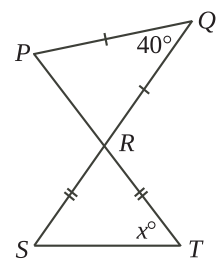
PART A

1. Determine the value of $\dfrac{\sqrt{25-16}}{\sqrt{25}-\sqrt{16}}$.

   {2008 Cayley #2}

2. In the diagram, PT and QS are straight lines intersecting at R such that $QP = QR$ and $RS = RT$. Determine the value of x.

   {2008 Cayley #8}



3. If $x+y+z=25$, $x+y=19$ and $y+z=18$, determine the value of y.

   {1998 Cayley #11}

4. The odd numbers from 5 to 21 are used to build a 3 by 3 magic square. (In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum.) If 5, 9 and 17 are placed as shown, what is the value of x?

   {2010 Cayley #16}

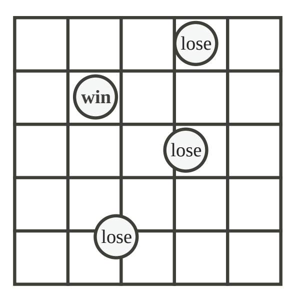
	5	
9		17
x		

5. What is the largest positive integer n that satisfies $n^{200} < 3^{500}$?

   {2010 Cayley #20}

6. A coin that is 8 cm in diameter is tossed onto a 5 by 5 grid of squares each having side length 10 cm. A coin is in a winning position if no part of it touches or crosses a grid line, otherwise it is in a losing position. Given that the coin lands in a random position so that no part of it is off the grid, what is the probability that it is in a winning position?

   {2010 Cayley #24}



PART B

1. (a) Determine the average of the integers $71, 72, 73, 74, 75$.

   (b) Suppose that $n, n+1, n+2, n+3, n+4$ are five consecutive integers.

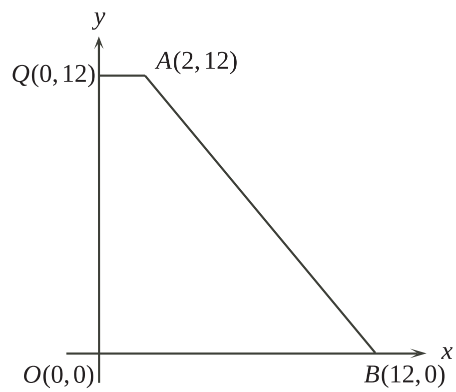
   (i) Determine a simplified expression for the sum of these five consecutive integers.

   (ii) If the average of these five consecutive integers is an odd integer, explain why n must be an odd integer.

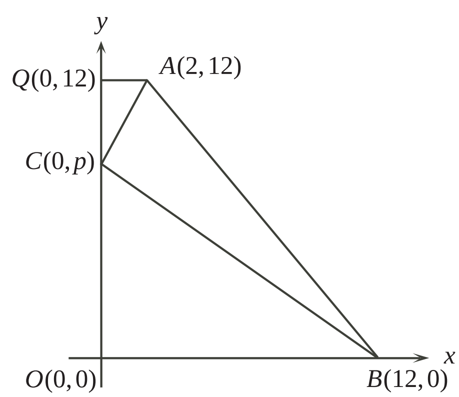
   (c) Six consecutive integers can be represented by $n, n+1, n+2, n+3, n+4, n+5$, where n is an integer. Explain why the average of six consecutive integers is never an integer.

{2010 Fryer #2}

2. (a) Quadrilateral $QABO$ is constructed as shown. Determine the area of $QABO$.



   (b) Point $C(0, p)$ lies on the y-axis between $Q(0, 12)$ and $O(0, 0)$ as shown. Determine an expression for the area of $\triangle COB$ in terms of p.



   (c) Determine an expression for the area of $\triangle QCA$ in terms of p.

   (d) If the area of $\triangle ABC$ is 27, determine the value of p.

{2010 Galois #2}

3. If m is a positive integer, the symbol $m!$ is used to represent the product of the integers from 1 to m. That is, $m! = m(m-1)(m-2)\dots(3)(2)(1)$. For example, $5! = 5(4)(3)(2)(1)$ or $5! = 120$. Some positive integers can be written in the form

$$n = a(1!) + b(2!) + c(3!) + d(4!) + e(5!).$$

In addition, each of the following conditions is satisfied:

- $a, b, c, d,$ and e are integers
- $0 \le a \le 1$
- $0 \le b \le 2$
- $0 \le c \le 3$
- $0 \le d \le 4$
- $0 \le e \le 5$.

(a) Determine the largest positive value of N that can be written in this form.

(b) Write $n = 653$ in this form.

(c) Prove that all integers n, where $0 \le n \le N$, can be written in this form.

(d) Determine the sum of all integers n that can be written in this form with $c = 0$.

{2009 Galois #4}