

**Canadian  
Mathematics  
Competition**

*An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario*

***2010 Pascal Contest***

*(Grade 9)*

**Thursday, February 25, 2010**

*Solutions*

1. In cents, the five given choices are 50, 90, 95, 101, and 115 cents.  
The differences between each of these and \$1.00 (or 100 cents), in cents, are

$$100 - 50 = 50 \quad 100 - 90 = 10 \quad 100 - 95 = 5 \quad 101 - 100 = 1 \quad 115 - 100 = 15$$

The difference between \$1.01 and \$1.00 is the smallest (1 cent), so \$1.01 is closest to \$1.00.

ANSWER: (D)

2. Using the correct order of operations,

$$\frac{(20 - 16) \times (12 + 8)}{4} = \frac{4 \times 20}{4} = \frac{80}{4} = 20$$

ANSWER: (C)

3. We divide the 750 mL of flour into portions of 250 mL. We do this by calculating  $750 \div 250 = 3$ .  
Therefore, 750 mL is three portions of 250 mL.

Since 50 mL of milk is required for each 250 mL of flour, then  $3 \times 50 = 150$  mL of milk is required in total.

ANSWER: (C)

4. There are 8 figures in total. Of these, 3 are triangles.

Therefore, the probability is  $\frac{3}{8}$ .

ANSWER: (A)

5. We simplify the left side and express it as a fraction with numerator 1:

$$\frac{1}{9} + \frac{1}{18} = \frac{2}{18} + \frac{1}{18} = \frac{3}{18} = \frac{1}{6}$$

Therefore, the number that replaces the  $\square$  is 6.

ANSWER: (C)

6. There are 16 horizontal segments on the perimeter. Each has length 1, so the horizontal segments contribute 16 to the perimeter.

There are 10 vertical segments on the perimeter. Each has length 1, so the vertical segments contribute 10 to the perimeter.

Therefore, the perimeter is  $10 + 16 = 26$ .

(We could arrive at this total instead by starting at a fixed point and travelling around the outside of the figure counting the number of segments.)

ANSWER: (E)

7. Since  $3^3 = 3 \times 3 \times 3 = 3 \times 9 = 27$ , then

$$\sqrt{3^3 + 3^3 + 3^3} = \sqrt{27 + 27 + 27} = \sqrt{81} = 9$$

ANSWER: (B)

8. The difference between the two given numbers is  $7.62 - 7.46 = 0.16$ .

This length of the number line is divided into 8 equal segments.

The length of each of these segments is thus  $0.16 \div 8 = 0.02$ .

Point  $P$  is three of these segments to the right of 7.46.

Thus, the number represented is  $7.46 + 3(0.02) = 7.46 + 0.06 = 7.52$ .

ANSWER: (E)

9. A 12 by 12 grid of squares will have 11 interior vertical lines and 11 interior horizontal lines. (In the given 4 by 4 example, there are 3 interior vertical lines and 3 interior horizontal lines.) Each of the 11 interior vertical lines intersects each of the 11 interior horizontal lines and creates an interior intersection point.

Thus, each interior vertical line accounts for 11 intersection points.

Therefore, the number of interior intersection points is  $11 \times 11 = 121$ .

ANSWER: (B)

10. Because the central angle for the interior sector “Less than 1 hour” is  $90^\circ$ , then the fraction of the students who do less than 1 hour of homework per day is  $\frac{90^\circ}{360^\circ} = \frac{1}{4}$ .

In other words, 25% of the students do less than 1 hour of homework per day.

Therefore,  $100\% - 25\% = 75\%$  of the students do at least 1 hour of homework per day.

ANSWER: (E)

11. *Solution 1*

Since there is more than 1 four-legged table, then there are at least 2 four-legged tables.

Since there are 23 legs in total, then there must be fewer than 6 four-legged tables, since 6 four-legged tables would have  $6 \times 4 = 24$  legs.

Thus, there are between 2 and 5 four-legged tables.

If there are 2 four-legged tables, then these tables account for  $2 \times 4 = 8$  legs, leaving  $23 - 8 = 15$  legs for the three-legged tables.

Since 15 is divisible by 3, then this must be the solution, so there are  $15 \div 3 = 5$  three-legged tables.

(We can check that if there are 3 or 4 four-legged tables, then the number of remaining legs is not divisible by 3, and if there are 5 four-legged tables, then there is only 1 three-legged table, which is not allowed.)

*Solution 2*

Since there is more than 1 table of each type, then there are at least 2 three-legged tables and 2 four-legged tables.

These tables account for  $2(3) + 2(4) = 14$  legs.

There are  $23 - 14 = 9$  more legs that need to be accounted for. These must come from a combination of three-legged and four-legged tables.

The only way to make 9 from 3s and 4s is to use three 3s.

Therefore, there are  $2 + 3 = 5$  three-legged tables and 2 four-legged tables.

ANSWER: (E)

12. *Solution 1*

The total area of the rectangle is  $3 \times 4 = 12$ .

The total area of the shaded regions equals the total area of the rectangle (12) minus the area of the unshaded region.

The unshaded region is a triangle with base of length 1 and height 4; the area of this region is  $\frac{1}{2}(1)(4) = 2$ .

Therefore, the total area of the shaded regions is  $12 - 2 = 10$ .

*Solution 2*

The shaded triangle on the left has base of length 2 and height of length 4, so has an area of  $\frac{1}{2}(2)(4) = 4$ .

The shaded triangle on the right has base of length 3 (at the top) and height of length 4, so has an area of  $\frac{1}{2}(3)(4) = 6$ .

Therefore, the total area of the shaded regions is  $4 + 6 = 10$ .

ANSWER: (C)

13. Since the ratio of boys to girls at Cayley H.S. is  $3 : 2$ , then  $\frac{3}{3+2} = \frac{3}{5}$  of the students at Cayley H.S. are boys.

Thus, there are  $\frac{3}{5}(400) = \frac{1200}{5} = 240$  boys at Cayley H.S.

Since the ratio of boys to girls at Fermat C.I. is  $2 : 3$ , then  $\frac{2}{2+3} = \frac{2}{5}$  of the students at Fermat C.I. are boys.

Thus, there are  $\frac{2}{5}(600) = \frac{1200}{5} = 240$  boys at Fermat C.I.

There are  $400 + 600 = 1000$  students in total at the two schools.

Of these,  $240 + 240 = 480$  are boys, and so the remaining  $1000 - 480 = 520$  students are girls.

Therefore, the overall ratio of boys to girls is  $480 : 520 = 48 : 52 = 12 : 13$ .

ANSWER: (B)

14. When the given net is folded, the face numbered 5 will be opposite the face numbered 1. Therefore, the remaining four faces share an edge with the face numbered 1, so the product of the numbers is  $2 \times 3 \times 4 \times 6 = 144$ .

ANSWER: (B)

15. The percentage 10% is equivalent to the fraction  $\frac{1}{10}$ .

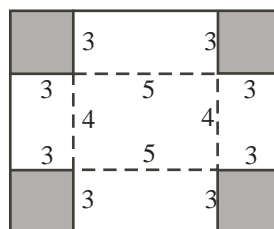
Therefore,  $t = \frac{1}{10}s$ , or  $s = 10t$ .

ANSWER: (D)

16. Since the base of the folded box measures 5 cm by 4 cm, then the area of the base of the box is  $5(4) = 20 \text{ cm}^2$ .

Since the volume of the box is  $60 \text{ cm}^3$  and the area of the base is  $20 \text{ cm}^2$ , then the height of the box is  $\frac{60}{20} = 3 \text{ cm}$ .

Therefore, each of the four identical squares has side length 3 cm, because the edges of these squares form the vertical edges of the box.



Therefore, the rectangular sheet measures  $3 + 5 + 3 = 11 \text{ cm}$  by  $3 + 4 + 3 = 10 \text{ cm}$ , and so has area  $11(10) = 110 \text{ cm}^2$ .

ANSWER: (B)

17. *Solution 1*

Since  $SUR$  is a straight line, then  $\angle RUV = 180^\circ - \angle SUV = 180^\circ - 120^\circ = 60^\circ$ .

Since  $PW$  and  $QX$  are parallel, then  $\angle RVW = \angle VTX = 112^\circ$ .

Since  $UVW$  is a straight line, then  $\angle RVU = 180^\circ - \angle RVW = 180^\circ - 112^\circ = 68^\circ$ .

Since the measures of the angles in a triangle add to  $180^\circ$ , then

$$\angle URV = 180^\circ - \angle RUV - \angle RVU = 180^\circ - 60^\circ - 68^\circ = 52^\circ$$

*Solution 2*

Since  $SUR$  is a straight line, then  $\angle RUV = 180^\circ - \angle SUV = 180^\circ - 120^\circ = 60^\circ$ .

Since  $PW$  and  $QX$  are parallel, then  $\angle RST = \angle RUV = 60^\circ$ .

Since  $STX$  is a straight line, then  $\angle RTS = 180^\circ - \angle VTX = 180^\circ - 112^\circ = 68^\circ$ .

Since the measures of the angles in a triangle add to  $180^\circ$ , then

$$\angle URV = \angle SRT = 180^\circ - \angle RST - \angle RTS = 180^\circ - 60^\circ - 68^\circ = 52^\circ$$

ANSWER: (A)

18. *Solution 1*

When Catherine adds 30 litres of gasoline, the tank goes from  $\frac{1}{8}$  full to  $\frac{3}{4}$  full.

Since  $\frac{3}{4} - \frac{1}{8} = \frac{6}{8} - \frac{1}{8} = \frac{5}{8}$ , then  $\frac{5}{8}$  of the capacity of the tank is 30 litres.

Thus,  $\frac{1}{8}$  of the capacity of the tank is  $30 \div 5 = 6$  litres. Also, the full capacity of the tank is  $8 \times 6 = 48$  litres.

To fill the remaining  $\frac{1}{4}$  of the tank, Catherine must add an additional  $\frac{1}{4} \times 48 = 12$  litres of gas. Because each litre costs \$1.38, it will cost  $12 \times \$1.38 = \$16.56$  to fill the rest of the tank.

*Solution 2*

Suppose that the capacity of the gas tank is  $x$  litres.

Starting with  $\frac{1}{8}$  of a tank, 30 litres of gas makes the tank  $\frac{3}{4}$  full, so  $\frac{1}{8}x + 30 = \frac{3}{4}x$  or  $\frac{5}{8}x = 30$  or  $x = 48$ .

The remaining capacity of the tank is  $\frac{1}{4}x = \frac{1}{4}(48) = 12$  litres.

At \$1.38 per litre, it will cost Catherine  $12 \times \$1.38 = \$16.56$  to fill the rest of the tank.

ANSWER: (C)

19. The area of a semi-circle with radius  $r$  is  $\frac{1}{2}\pi r^2$  so the area of a semi-circle with diameter  $d$  is  $\frac{1}{2}\pi(\frac{1}{2}d)^2 = \frac{1}{8}\pi d^2$ .

The semicircles with diameters  $UV$ ,  $VW$ ,  $WX$ ,  $XY$ , and  $YZ$  each have equal diameter and thus equal area. The area of each of these semicircles is  $\frac{1}{8}\pi(5^2) = \frac{25}{8}\pi$ .

The large semicircle has diameter  $UZ = 5(5) = 25$ , so has area  $\frac{1}{8}\pi(25^2) = \frac{625}{8}\pi$ .

The shaded area equals the area of the large semicircle, minus the area of two small semicircles, plus the area of three small semicircles, which equals the area of the large semicircle plus the area of one small semicircle.

Therefore, the shaded area equals  $\frac{625}{8}\pi + \frac{25}{8}\pi = \frac{650}{8}\pi = \frac{325}{4}\pi$ .

ANSWER: (A)

20. The sum of the odd numbers from 5 to 21 is

$$5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 = 117$$

Therefore, the sum of the numbers in any row is one-third of this total, or 39.

This means as well that the sum of the numbers in any column or diagonal is also 39.

Since the numbers in the middle row add to 39, then the number in the centre square is  $39 - 9 - 17 = 13$ .

Since the numbers in the middle column add to 39, then the number in the middle square in the bottom row is  $39 - 5 - 13 = 21$ .

	5	
9	13	17
$x$	21	

Since the numbers in the bottom row add to 39, then the number in the bottom right square is  $39 - 21 - x = 18 - x$ .

Since the numbers in the bottom left to top right diagonal add to 39, then the number in the top right square is  $39 - 13 - x = 26 - x$ .

Since the numbers in the rightmost column add to 39, then  $(26 - x) + 17 + (18 - x) = 39$  or  $61 - 2x = 39$  or  $2x = 22$ , and so  $x = 11$ .

We can complete the magic square as follows:

19	5	15
9	13	17
11	21	7

ANSWER: (B)

21. We label the numbers in the empty boxes as  $y$  and  $z$ , so the numbers in the boxes are thus  $8, y, z, 26, x$ .

Since the average of  $z$  and  $x$  is 26, then  $x + z = 2(26) = 52$  or  $z = 52 - x$ .

We rewrite the list as  $8, y, 52 - x, 26, x$ .

Since the average of 26 and  $y$  is  $52 - x$ , then  $26 + y = 2(52 - x)$  or  $y = 104 - 26 - 2x = 78 - 2x$ .

We rewrite the list as  $8, 78 - 2x, 52 - x, 26, x$ .

Since the average of 8 and  $52 - x$  is  $78 - 2x$ , then

$$\begin{aligned} 8 + (52 - x) &= 2(78 - 2x) \\ 60 - x &= 156 - 4x \\ 3x &= 96 \\ x &= 32 \end{aligned}$$

Therefore,  $x = 32$ .

ANSWER: (D)

22. Since  $JKLM$  is a rectangle, then the angles at  $J$  and  $K$  are each  $90^\circ$ , so each of  $\triangle SJP$  and  $\triangle QKP$  is right-angled.

By the Pythagorean Theorem in  $\triangle SJP$ , we have

$$SP^2 = JS^2 + JP^2 = 52^2 + 39^2 = 2704 + 1521 = 4225$$

Since  $SP > 0$ , then  $SP = \sqrt{4225} = 65$ .

Since  $PQRS$  is a rhombus, then  $PQ = PS = 65$ .

By the Pythagorean Theorem in  $\triangle QKP$ , we have

$$KP^2 = PQ^2 - KQ^2 = 65^2 - 25^2 = 4225 - 625 = 3600$$

Since  $KP > 0$ , then  $KP = \sqrt{3600} = 60$ .

(Instead of using the Pythagorean Theorem, we could note instead that  $\triangle SJP$  is a scaled-up version of a 3-4-5 right-angled triangle and that  $\triangle QKP$  is a scaled-up version of a 5-12-13 right-angled triangle. This would allow us to use the known ratios of side lengths to calculate the missing side length.)

Since  $KQ$  and  $PZ$  are parallel and  $PK$  and  $WQ$  are parallel, then  $PKQW$  is a rectangle, and

so  $PW = KQ = 25$ .

Similarly,  $JPZS$  is a rectangle and so  $PZ = JS = 52$ .

Thus,  $WZ = PZ - PW = 52 - 25 = 27$ .

Also,  $SYRM$  is a rectangle. Since  $JM$  and  $KL$  are parallel ( $JKLM$  is a rectangle),  $JK$  and  $ML$  are parallel, and  $PQ$  and  $SR$  are parallel ( $PQRS$  is a rhombus), then  $\angle MSR = \angle KQP$  and  $\angle SRM = \angle QPK$ .

Since  $\triangle SMR$  and  $\triangle QKP$  have two equal angles, then their third angles must be equal too. Thus, the triangles have the same proportions. Since the hypotenuses of the triangles are equal, then the triangles must in fact be exactly the same size; that is, the lengths of the corresponding sides must be equal. (We say that  $\triangle SMR$  is congruent to  $\triangle QKP$  by “angle-side-angle”.)

In particular,  $MR = KP = 60$ .

Thus,  $ZY = SY - SZ = MR - JP = 60 - 39 = 21$ .

Therefore, the perimeter of rectangle  $WXYZ$  is  $2(21) + 2(27) = 96$ .

ANSWER: (D)

23. First, we note that  $2010 = 10(201) = 2(5)(3)(67)$  and so  $2010^2 = 2^2 3^2 5^2 67^2$ .

Consider  $N$  consecutive four-digit positive integers.

For the product of these  $N$  integers to be divisible by  $2010^2$ , it must be the case that two different integers are divisible by 67 (which would mean that there are at least 68 integers in the list) or one of the integers is divisible by  $67^2$ .

Since we want to minimize  $N$  (and indeed because none of the answer choices is at least 68), we look for a list of integers in which one is divisible by  $67^2 = 4489$ .

Since the integers must all be four-digit integers, then the only multiples of 4489 that we must consider are 4489 and 8978.

First, we consider a list of  $N$  consecutive integers including 4489.

Since the product of these integers must have 2 factors of 5 and no single integer within 10 of 4489 has a factor of 25, then the list must include two integers that are multiples of 5. To minimize the number of integers in the list, we try to include 4485 and 4490.

Thus our candidate list is 4485, 4486, 4487, 4488, 4489, 4490.

The product of these integers includes 2 factors of 67 (in 4489), 2 factors of 5 (in 4485 and 4490), 2 factors of 2 (in 4486 and 4488), and 2 factors of 3 (since each of 4485 and 4488 is divisible by 3). Thus, the product of these 6 integers is divisible by  $2010^2$ .

Therefore, the shortest possible list including 4489 has length 6.

Next, we consider a list of  $N$  consecutive integers including 8978.

Here, there is a nearby integer containing 2 factors of 5, namely 8975.

So we start with the list 8975, 8976, 8977, 8978 and check to see if it has the required property. The product of these integers includes 2 factors of 67 (in 8978), 2 factors of 5 (in 8975), and 2 factors of 2 (in 8976). However, the only integer in this list divisible by 3 is 8976, which has only 1 factor of 3.

To include a second factor of 3, we must include a second multiple of 3 in the list. Thus, we extend the list by one number to 8979.

Therefore, the product of the numbers in the list 8975, 8976, 8977, 8978, 8979 is a multiple of  $2010^2$ . The length of this list is 5.

Thus, the smallest possible value of  $N$  is 5.

(Note that a quick way to test if an integer is divisible by 3 is to add its digit and see if this total is divisible by 3. For example, the sum of the digits of 8979 is 33; since 33 is a multiple of 3, then 8979 is a multiple of 3.)

ANSWER: (A)

24. We label the terms  $x_1, x_2, x_3, \dots, x_{2009}, x_{2010}$ .  
Suppose that  $S$  is the sum of the odd-numbered terms in the sequence; that is,

$$S = x_1 + x_3 + x_5 + \cdots + x_{2007} + x_{2009}$$

We know that the sum of all of the terms is 5307; that is,

$$x_1 + x_2 + x_3 + \cdots + x_{2009} + x_{2010} = 5307$$

Next, we pair up the terms: each odd-numbered term with the following even-numbered term. That is, we pair the first term with the second, the third term with the fourth, and so on, until we pair the 2009th term with the 2010th term. There are 1005 such pairs.

In each pair, the even-numbered term is one bigger than the odd-numbered term. That is,  $x_2 - x_1 = 1$ ,  $x_4 - x_3 = 1$ , and so on.

Therefore, the sum of the even-numbered terms is 1005 greater than the sum of the odd-numbered terms. Thus, the sum of the even-numbered terms is  $S + 1005$ .

Since the sum of all of the terms equals the sum of the odd-numbered terms plus the sum of the even-numbered terms, then  $S + (S + 1005) = 5307$  or  $2S = 4302$  or  $S = 2151$ .

Thus, the required sum is 2151.

ANSWER: (C)

25. Before we answer the given question, we determine the number of ways of choosing 3 objects from 5 objects and the number of ways of choosing 2 objects from 5 objects.

Consider 5 objects labelled B, C, D, E, F.

The possible pairs are: BC, BD, BE, BF, CD, CE, CF, DE, DF, EF. There are 10 such pairs.

The possible triples are: DEF, CEF, CDF, CDE, BEF, BDF, BDE, BCF, BCE, BCD. There are 10 such triples.

(Can you see why there are the same number of pairs and triples?)

Label the six teams A, B, C, D, E, F.

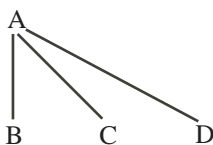
We start by considering team A.

Team A plays 3 games, so we must choose 3 of the remaining 5 teams for A to play. As we saw above, there are 10 ways to do this.

Without loss of generality, we pick one of these sets of 3 teams for A to play, say A plays B, C and D.

We keep track of everything by drawing diagrams, joining the teams that play each other with a line.

Thus far, we have



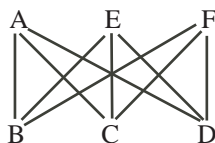
There are two possible cases now – either none of B, C and D play each other, or at least one pair of B, C, D plays each other.

Case 1: None of the teams that play A play each other

In the configuration above, each of B, C and D play two more games. They already play A and cannot play each other, so they must each play E and F.

This gives





No further choices are possible.

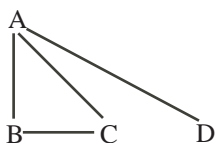
There are 10 possible schedules in this type of configuration. These 10 combinations come from choosing the 3 teams that play A.

Case 2: Some of the teams that play A play each other

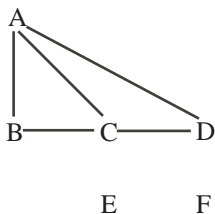
Here, at least one pair of the teams that play A play each other.

Given the teams B, C and D playing A, there are 3 possible pairs (BC, BD, CD).

We pick one of these pairs, say BC. (This gives  $10 \times 3 = 30$  configurations so far.)



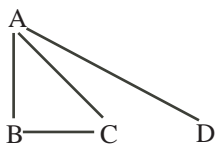
It is now not possible for B or C to also play D. If it was the case that C, say, played D, then we would have the configuration



Here, A and C have each played 3 games and B and D have each played 2 games. Teams E and F are unaccounted for thus far. They cannot both play 3 games in this configuration as the possible opponents for E are B, D and F, and the possible opponents for F are B, D and E, with the “B” and “D” possibilities only to be used once.

A similar argument shows that B cannot play D.

Thus, B or C cannot also play D. So we have the configuration

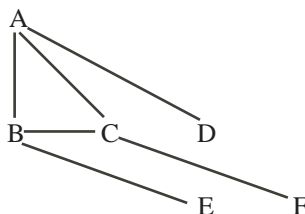


Here, A has played 3 games, B and C have each played 2 games, and D has played 1 game.

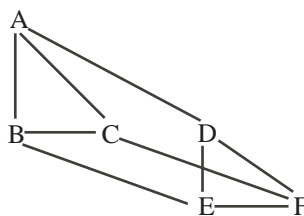
B and C must play 1 more game and cannot play D or A.

They must play E and F in some order. There are 2 possible ways to assign these games (BE and CF, or BF and CE.) This gives  $30 \times 2 = 60$  configurations so far.

Suppose that B plays E and C plays F.



So far, A, B and C each play 3 games and E, F and D each play 1 game.  
The only way to complete the configuration is to join D, E and F.



Therefore, there are 60 possible schedules in this case.

In total, there are  $10 + 60 = 70$  possible schedules.

ANSWER: (E)

