# 2010 Galois Contest (Grade 10)
## Friday, April 9, 2010

1. Emily's old showerhead used 18 L of water per minute.
   She installs a new showerhead that uses 13 L per minute.

   (a) When Emily takes a bath, she uses 260 L of water. Using the new showerhead, what length of shower, in minutes, uses 260 L of water?

   (b) How much *less* water is used for a 10 minute shower with the new showerhead than with the old showerhead?

   (c) Emily is charged 8 cents per 100 L of water that she uses. Using the new showerhead instead of the old showerhead saves water and so saves Emily money. How much money does Emily *save* in water costs for a 15 minute shower?

   (d) How many minutes of showering, using the new showerhead, will it take for Emily to have saved $30 in water costs?

2. (a) Quadrilateral $QABO$ is constructed as shown. Determine the area of $QABO$.
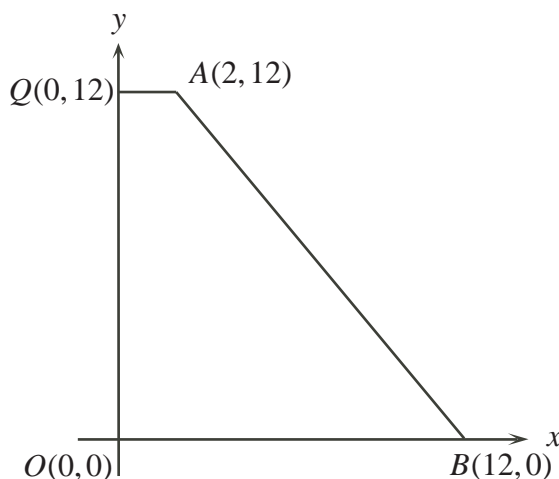
   (b) Point $C(0, p)$ lies on the y-axis between $Q(0, 12)$ and $O(0, 0)$ as shown. Determine an expression for the area of $\triangle COB$ in terms of p.

   (c) Determine an expression for the area of $\triangle QCA$ in terms of p.

   (d) If the area of $\triangle ABC$ is 27, determine the value of p.
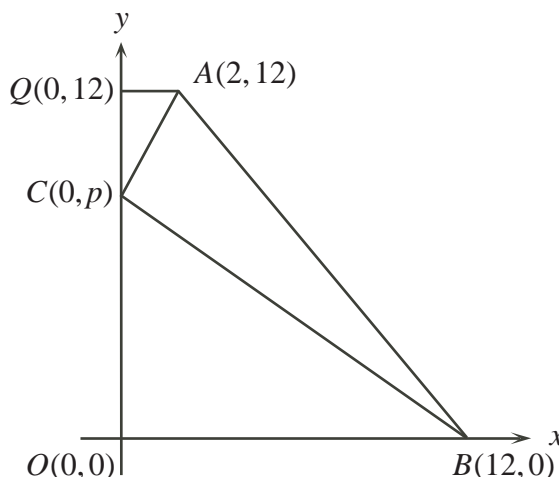
3. (a) Solve the system of equations algebraically for (x, y):
$$x + y = 42$$
$$x - y = 10$$

(b) Suppose that p is an even integer and that q is an odd integer. Explain why the system of equations
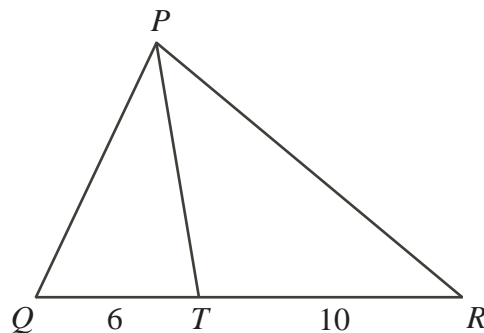$$x + y = p$$
$$x - y = q$$

has no positive integer solutions (x, y).

(c) Determine all pairs of positive integers (x, y) that satisfy the equation $x^2 - y^2 = 420$.

4. (a) In $\triangle PQR$, point T is on side QR such that $QT = 6$ and $TR = 10$. Explain why the ratio of the area of $\triangle PQT$ to the area of $\triangle PTR$ is $3 : 5$.

(b) In $\triangle ABC$, point D is the midpoint of side BC. Point E is on AC such that $AE : EC = 1 : 2$. Point F is on AD such that $AF : FD = 3 : 1$. If the area of $\triangle DEF$ is 17, determine the area of $\triangle ABC$.
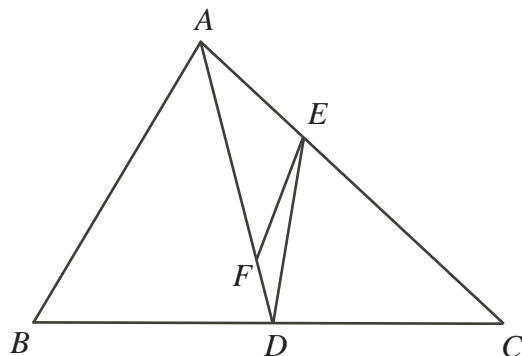
(c) In the diagram, points X, Y and Z are on the sides of $\triangle UVW$, as shown. Line segments UY, VZ and WX intersect at P. Point Y is on VW such that $VY : YW = 4 : 3$. If $\triangle PYW$ has an area of 30 and $\triangle PZW$ has an area of 35, determine the area of $\triangle UXP$.