Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 18, 2009

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Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $3 + 3^3$ is

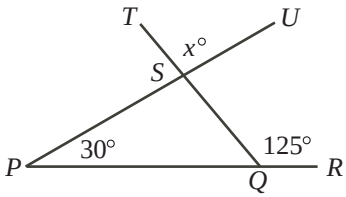
   (A) 12 **(B)** 18 **(C)** 216 **(D)** 30 **(E)** 36

2. If $3 \times 2 + 8 = \nabla + 5$, then ∇ equals

   (A) 14 **(B)** 25 **(C)** 19 **(D)** 17 **(E)** 9

3. In the diagram, PQR, QST and PSU are straight lines. The value of x is

   (A) 75 **(B)** 85 **(C)** 95 **(D)** 125 **(E)** 155



4. If $w = 4$, $x = 9$, and $z = 25$ then $\sqrt{\dfrac{w}{x}} + \sqrt{\dfrac{x}{z}}$ equals

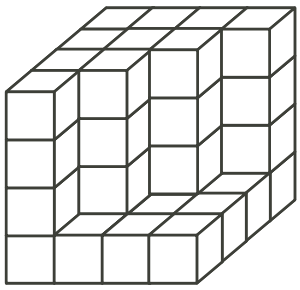
   (A) $\dfrac{5}{8}$ **(B)** $\dfrac{19}{15}$ **(C)** $\dfrac{77}{225}$ **(D)** $\dfrac{181}{225}$ **(E)** $\dfrac{2}{5}$

5. $1 - 4(3-1)^{-1}$ is equal to

   (A) -1 **(B)** $-\frac{3}{2}$ **(C)** 9 **(D)** 6 **(E)** $\frac{11}{3}$

6. Sixty-four identical cubes are stacked in a $4 \times 4 \times 4$ arrangement and then some of the cubes are removed from the front as shown. No cube hidden from sight has been removed. How many cubes remain in the arrangement?

   (A) 46 **(B)** 40 **(C)** 52 **(D)** 55 **(E)** 49



7. If $n > 0$ and $\sqrt{n^2 + n^2 + n^2 + n^2} = 64$, then n equals

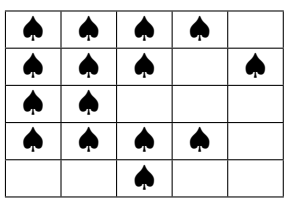
   (A) $\sqrt{8}$ **(B)** 16 **(C)** 4 **(D)** 32 **(E)** $\sqrt{2}$

8. Gavin has a collection of 50 songs that are each 3 minutes in length and 50 songs that are each 5 minutes in length. What is the maximum number of songs from his collection that he can play in 3 hours?

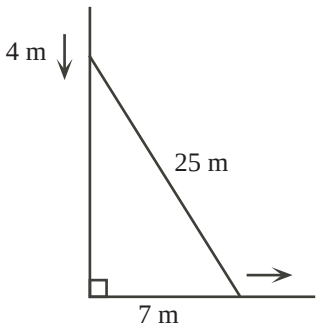
   (A) 100 **(B)** 36 **(C)** 56 **(D)** 60 **(E)** 45

9. In the diagram, any ♠ may be moved to any unoccupied space. What is the smallest number of ♠'s that must be moved so that each row and each column contains three ♠'s?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5



10. Judi leans a 25 m ladder against a vertical wall with the bottom of the ladder 7 m from the wall. (Please note that Judi is very strong – don't try this at home!) As she pulls the bottom of the ladder away from the wall, the top of the ladder slides 4 m down the wall. How far did she pull the bottom of the ladder from its original position?



(A) 4 m **(B)** 11 m **(C)** 2 m
(D) 13 m **(E)** 8 m

Part B: Each correct answer is worth 6.

11. Suppose m and n are positive integers with $m < n$. The value of $\dfrac{m+3}{n+3}$ will be

(A) equal to 1
(B) equal to 3
(C) less than the value of $\dfrac{m}{n}$
(D) greater than the value of $\dfrac{m}{n}$
(E) equal to the value of $\dfrac{m}{n}$

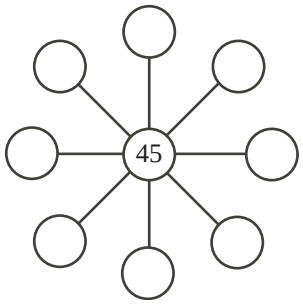
12. How many four-digit integers between 5000 and 6000 are there for which the thousands digit equals the sum of the other three digits? (The thousands digit of 5124 is 5.)

(A) 5 **(B)** 15 **(C)** 21 **(D)** 30 **(E)** 12

13. The number of integers x for which the value of $\dfrac{-6}{x+1}$ is an integer is

(A) 8 **(B)** 9 **(C)** 2 **(D)** 6 **(E)** 7

14. Different positive integers can be written in the eight empty circles so that the product of any three integers in a straight line is 3240. What is the largest possible sum of the eight numbers surrounding 45?

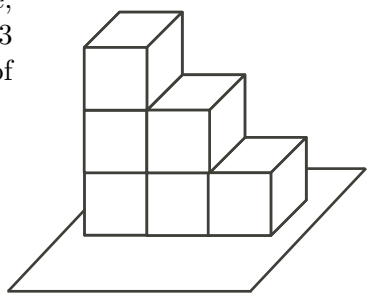


(A) 139 **(B)** 211 **(C)** 156
(D) 159 **(E)** 160

15. On Monday, 10% of the students at Dunkley S.S. were absent and 90% were present. On Tuesday, 10% of those who were absent on Monday were present and the rest of those absent on Monday were still absent. Also, 10% of those who were present on Monday were absent and the rest of those present on Monday were still present. What percentage of the students at Dunkley S.S. were present on Tuesday?

(A) 81% **(B)** 82% **(C)** 90% **(D)** 91% **(E)** 99%

16. Six dice are stacked on the floor as shown. On each die, the 1 is opposite the 6, the 2 is opposite the 5, and the 3 is opposite the 4. What is the maximum possible sum of numbers on the 21 visible faces?



(A) 69 **(B)** 88 **(C)** 89
(D) 91 **(E)** 96

17. In the diagram, the perimeter of the semicircular region is 20. (The perimeter includes both the semicircular arc and the diameter.) The area of the region is closest to

(A) 36.6 **(B)** 23.8 **(C)** 49.3
(D) 51.6 **(E)** 26.7

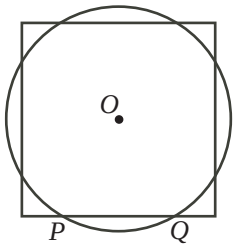
18. On Monday, Hank drove to work at an average speed of 70 km/h and arrived 1 minute late. On Tuesday, he left at the same time and took the same route. This time he drove at an average speed of 75 km/h and arrived 1 minute early. How long is his route to work?

(A) 30 km **(B)** 35 km **(C)** 45 km **(D)** 50 km **(E)** 60 km

19. If $2^x = 15$ and $15^y = 32$, the value of xy is

(A) 5 **(B)** 8 **(C)** 16 **(D)** 6 **(E)** 4

20. In the diagram, the circle and the square have the same centre O and equal areas. The circle has radius 1 and intersects one side of the square at P and Q. What is the length of PQ?



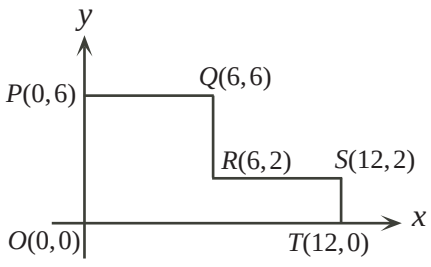
(A) $\sqrt{4-\pi}$ **(B)** 1 **(C)** $\sqrt{2}$
(D) $2-\sqrt{\pi}$ **(E)** $4-\sqrt{\pi}$

Part C: Each correct answer is worth 8.

21. At Matilda's birthday party, the ratio of people who ate ice cream to people who ate cake was 3 : 2. People who ate both ice cream and cake were included in both categories. If 120 people were at the party, what is the maximum number of people who could have eaten both ice cream and cake?

(A) 24 **(B)** 30 **(C)** 48 **(D)** 80 **(E)** 72

22. In the diagram, two straight lines are to be drawn through $O(0,0)$ so that the lines divide the figure $OPQRST$ into 3 pieces of equal area. The sum of the slopes of the lines will be



(A) $\frac{35}{24}$ **(B)** $\frac{7}{6}$ **(C)** $\frac{5}{4}$
(D) $\frac{4}{3}$ **(E)** $\frac{11}{8}$

23. Suppose that a, b, c, and d are positive integers that satisfy the equations

$$\begin{aligned} ab + cd &= 38 \\ ac + bd &= 34 \\ ad + bc &= 43 \end{aligned}$$

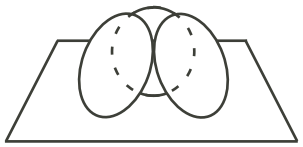
What is the value of $a + b + c + d$?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

24. Starting with the input (m, n), Machine A gives the output (n, m).
Starting with the input (m, n), Machine B gives the output $(m + 3n, n)$.
Starting with the input (m, n), Machine C gives the output $(m - 2n, n)$.
Natalie starts with the pair $(0, 1)$ and inputs it into one of the machines. She takes the output and inputs it into any one of the machines. She continues to take the output that she receives and inputs it into any one of the machines. (For example, starting with $(0, 1)$, she could use machines B, B, A, C, B in that order to obtain the output $(7, 6)$.) Which of the following pairs is impossible for her to obtain after repeating this process any number of times?

(A) (2009,1016) **(B)** (2009,1004) **(C)** (2009,1002)
(D) (2009,1008) **(E)** (2009,1032)

25. In the diagram, three circles of radius 10 are tangent to each other and to a plane in three-dimensional space. Each of the circles is inclined at 45° to the plane. There are three points where the circles touch each other. These three points lie on a circle parallel to the plane. The radius of this circle is closest to



(A) 6.9 **(B)** 7.1 **(C)** 7.3
(D) 7.5 **(E)** 7.7

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For students...

Thank you for writing the 2009 Fermat Contest!
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