Canadian
Mathematics
Competition
An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

2008 Pascal Contest

(Grade 9)

Tuesday, February 19, 2008

Solutions

1. Calculating, $\dfrac{2+3+4}{2\times 3\times 4}=\dfrac{9}{24}=\dfrac{3}{8}$.

   Answer: (E)

2. Since $3x-9=12$, then $3x=12+9=21$.
   Since $3x=21$, then $6x=2(3x)=2(21)=42$.
   (Note that we did not need to determine the value of x.)

   Answer: (A)

3. Calculating, $\sqrt{5^2-4^2}=\sqrt{25-16}=\sqrt{9}=3$.

   Answer: (B)

4. *Solution 1*
   Since $JLMR$ is a rectangle and $JR=2$, then $LM=2$.
   Similarly, since $JL=8$, then $RM=8$.
   Since $RM=8$ and $RQ=3$, then $QM=8-3=5$.
   Since $KLMQ$ is a rectangle with $QM=5$ and $LM=2$, its area is $5(2)=10$.

   Solution 2
   Since $JL=8$ and $JR=2$, then the area of rectangle $JLMR$ is $2(8)=16$.
   Since $RQ=3$ and $JR=2$, then the area of rectangle $JKQR$ is $2(3)=6$.
   The area of rectangle $KLMQ$ is the difference between these areas, or $16-6=10$.

   Answer: (C)

5. Since $x=12$ and $y=-6$, then

$$\frac{3x+y}{x-y}=\frac{3(12)+(-6)}{12-(-6)}=\frac{30}{18}=\frac{5}{3}$$

   Answer: (C)

6. *Solution 1*
   Since $\angle PQS$ is an exterior angle of $\triangle QRS$, then $\angle PQS=\angle QRS+\angle QSR$, so $136^\circ=x^\circ+64^\circ$ or $x=136-64=72$.

   Solution 2
   Since $\angle PQS=136^\circ$, then $\angle RQS=180^\circ-\angle PQS=180^\circ-136^\circ=44^\circ$.
   Since the sum of the angles in $\triangle QRS$ is 180°, then $44^\circ+64^\circ+x^\circ=180^\circ$ or $x=180-44-64=72$.

   Answer: (A)

7. In total, there are $5+6+7+8=26$ jelly beans in the bag.
   Since there are 8 blue jelly beans, the probability of selecting a blue jelly bean is $\frac{8}{26}=\frac{4}{13}$.

   Answer: (D)

8. Since Olave sold 108 apples in 6 hours, then she sold $108\div 6=18$ apples in one hour.
   A time period of 1 hour and 30 minutes is equivalent to 1.5 hours.
   Therefore, Olave will sell $1.5\times 18=27$ apples in 1 hour and 30 minutes.

   Answer: (A)

9. Since the length of the rectangular grid is 10 and the grid is 5 squares wide, then the side length of each square in the grid is $10 \div 5 = 2$.
There are 4 horizontal wires, each of length 10, which thus have a total length of $4 \times 10 = 40$.
Since the side length of each square is 2 and the rectangular grid is 3 squares high, then the length of each vertical wire is $3 \times 2 = 6$.
Since there are 6 vertical wires, the total length of the vertical wires is $6 \times 6 = 36$.
Therefore, the total length of wire is $40 + 36 = 76$.

ANSWER: (E)

10. *Solution 1*
Since Q is at 46 and P is at -14, then the distance along the number line from P to Q is $46 - (-14) = 60$.
Since S is three-quarters of the way from P to Q, then S is at $-14 + \frac{3}{4}(60) = -14 + 45 = 31$.
Since T is one-third of the way from P to Q, then T is at $-14 + \frac{1}{3}(60) = -14 + 20 = 6$.
Thus, the distance along the number line from T to S is $31 - 6 = 25$.

Solution 2
Since Q is at 46 and P is at -14, then the distance along the number line from P to Q is $46 - (-14) = 60$.
Since S is three-quarters of the way from P to Q and T is one-third of the way from P to Q, then the distance from T to S is $60\left(\frac{3}{4} - \frac{1}{3}\right) = 60\left(\frac{9}{12} - \frac{4}{12}\right) = 60\left(\frac{5}{12}\right) = 25$.

ANSWER: (D)

11. In total, $30 + 20 = 50$ students wrote the Pascal Contest at Mathville Junior High.
Since 30% (or $\frac{3}{10}$) of the boys won certificates and 40% (or $\frac{4}{10}$) of the girls won certificates, then the total number of certificates awarded was $\frac{3}{10}(30) + \frac{4}{10}(20) = 9 + 8 = 17$.
Therefore, 17 of 50 participating students won certificates. In other words, $\frac{17}{50} \times 100\% = 34\%$ of the participating students won certificates.

ANSWER: (A)

12. Since the perimeter of the rectangle is 56, then

$$\begin{aligned} 2(x+4) + 2(x-2) &= 56 \\ 2x + 8 + 2x - 4 &= 56 \\ 4x + 4 &= 56 \\ 4x &= 52 \\ x &= 13 \end{aligned}$$

Therefore, the rectangle is $x + 4 = 17$ by $x - 2 = 11$, so has area $17(11) = 187$.

ANSWER: (B)

13. Using exponent rules, $2^3 \times 2^2 \times 3^3 \times 3^2 = 2^{3+2} \times 3^{3+2} = 2^5 \times 3^5 = (2 \times 3)^5 = 6^5$.

ANSWER: (A)

14. *Solution 1*
The wording of the problem tells us that $a + b + c + d + e + f$ must be the same no matter what numbers abc and def are chosen that satisfy the conditions.
An example that works is $889 + 111 = 1000$.
In this case, $a + b + c + d + e + f = 8 + 8 + 9 + 1 + 1 + 1 = 28$, so this must always be the value.

Solution 2
Consider performing this "long addition" by hand.
Consider first the units column.
Since $c+f$ ends in a 0, then $c+f=0$ or $c+f=10$. The value of $c+f$ cannot be 20 or more, as c and f are digits.
Since none of the digits is 0, we cannot have $c+f=0+0$ so $c+f=10$. (This means that we "carry" a 1 to the tens column.)
Since the result in the tens column is 0 and there is a 1 carried into this column, then $b+e$ ends in a 9, so we must have $b+e=9$. (Since b and e are digits, $b+e$ cannot be 19 or more.)
In the tens column, we thus have $b+e=9$ plus the carry of 1, so the resulting digit in the tens column is 0, with a 1 carried to the hundreds column.
Using a similar analysis in the hundreds column to that in the tens column, we must have $a+d=9$.
Therefore, $a+b+c+d+e+f=(a+d)+(b+e)+(c+f)=9+9+10=28$.

Answer: (D)

15. Each of $\triangle PSQ$ and $\triangle RSQ$ is right-angled at S, so we can use the Pythagorean Theorem in both triangles.
In $\triangle RSQ$, we have $QS^2=QR^2-SR^2=25^2-20^2=625-400=225$, so $QS=\sqrt{225}=15$ since $QS>0$.
In $\triangle PSQ$, we have $PQ^2=PS^2+QS^2=8^2+225=64+225=289$, so $PQ=\sqrt{289}=17$ since $PQ>0$.
Therefore, the perimeter of $\triangle PQR$ is $PQ+QR+RP=17+25+(20+8)=70$.

Answer: (E)

16. Suppose the radius of the circle is r cm.
Then the area M is πr^2 cm^2 and the circumference N is $2\pi r$ cm.
Thus, $\dfrac{\pi r^2}{2\pi r}=20$ or $\dfrac{r}{2}=20$ or $r=40$.

Answer: (C)

17. *Solution 1*
The large cube has a total surface area of 5400 cm^2 and its surface is made up of 6 identical square faces. Thus, the area of each face, in square centimetres, is $5400\div 6=900$.
Because each face is square, the side length of each face is $\sqrt{900}=30$ cm.
Therefore, each edge of the cube has length 30 cm and so the large cube has a volume of $30^3=27\,000$ cm^3.
Because the large cube is cut into small cubes each having volume 216 cm^3, then the number of small cubes equals $27\,000\div 216=125$.

Solution 2
Since the large cube has 6 square faces of equal area and the total surface area of the cube is 5400 cm^2, then the surface area of each face is $5400\div 6=900$ cm^2.
Since each face is square, then the side length of each square face of the cube is $\sqrt{900}=30$ cm, and so the edge length of the cube is 30 cm.
Since each smaller cube has a volume of 216 cm^3, then the side length of each smaller cube is $\sqrt[3]{216}=6$ cm.
Since the side length of the large cube is 30 cm and the side length of each smaller cube is 6 cm, then $30\div 6=5$ smaller cubes fit along each edge of the large cube.
Thus, the large cube is made up of $5^3=125$ smaller cubes.

Answer: (B)

18. *Solution 1*

Alex has 265 cents in total.
Since 265 is not divisible by 10, Alex cannot have only dimes, so must have at least 1 quarter.
If Alex has 1 quarter, then he has $265 - 25 = 240$ cents in dimes, so 24 dimes.
Alex cannot have 2 quarters, since $265 - 2(25) = 215$ is not divisible by 10.
If Alex has 3 quarters, then he has $265 - 3(25) = 190$ cents in dimes, so 19 dimes.
Continuing this argument, we can see that Alex cannot have an even number of quarters, since the total value in cents of these quarters would end in a 0, making the total value of the dimes end in a 5, which is not possible.
If Alex has 5 quarters, then he has $265 - 5(25) = 140$ cents in dimes, so 14 dimes.
If Alex has 7 quarters, then he has $265 - 7(25) = 90$ cents in dimes, so 9 dimes.
If Alex has 9 quarters, then he has $265 - 9(25) = 40$ cents in dimes, so 4 dimes.
If Alex has more than 9 quarters, then he will have even fewer than 4 dimes, so we do not need to investigate any more possibilities since we are told that Alex has more dimes than quarters.
So the possibilities for the total number of coins that Alex has are $1 + 24 = 25$, $3 + 19 = 22$, $5 + 14 = 19$, and $7 + 9 = 16$.
Therefore, the smallest number of coins that Alex could have is 16.
(Notice that each time we increase the number of quarters above, we are in effect exchanging 2 quarters (worth 50 cents) for 5 dimes (also worth 50 cents).)

Solution 2

Suppose that Alex has d dimes and q quarters, where d and q are non-negative integers.
Since Alex has \$2.65, then $10d + 25q = 265$ or $2d + 5q = 53$.
Since the right side is odd, then the left side must be odd, so $5q$ must be odd, so q must be odd.
If $q \geq 11$, then $5q \geq 55$, which is too large.
Therefore, $q < 11$, leaving $q = 1, 3, 5, 7, 9$ which give $d = 24, 19, 14, 9, 4$.
The solution with $d > q$ and $d + q$ smallest is $q = 7$ and $d = 9$, giving 16 coins in total.

ANSWER: (B)

19. From the definition, the first and second digits of an upright integer automatically determine the third digit, since it is the sum of the first two digits.
Consider first those upright integers beginning with 1.
These are 101, 112, 123, 134, 145, 156, 167, 178, and 189, since $1 + 0 = 1$, $1 + 1 = 2$, and so on. (The second digit cannot be 9, otherwise the last "digit" would be $1 + 9 = 10$, which is impossible.) There are 9 such numbers.
Beginning with 2, the upright integers are 202, 213, 224, 235, 246, 257, 268, and 279. There are 8 of them.
We can continue the pattern and determine the numbers of the upright integers beginning with 3, 4, 5, 6, 7, 8, and 9 to be 7, 6, 5, 4, 3, 2, and 1.
Therefore, there are $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ positive 3-digit upright integers.

ANSWER: (D)

20. The sum of the six given integers is $1867 + 1993 + 2019 + 2025 + 2109 + 2121 = 12134$.
The four of these integers that have a mean of 2008 must have a sum of $4(2008) = 8032$. (We do not know which integers they are, but we do not actually need to know.)
Thus, the sum of the remaining two integers must be $12134 - 8032 = 4102$.
Therefore, the mean of the remaining two integers is $\frac{4102}{2} = 2051$.

(We can verify that 1867, 2019, 2025 and 2121 do actually have a mean of 2008, and that 1993 and 2109 have a mean of 2051.)

Answer: (D)

21. The maximum possible value of $\dfrac{p}{q}$ is when p is as large as possible (that is, 10) and q is as small as possible (that is, 12). Thus, the maximum possible value of $\dfrac{p}{q}$ is $\dfrac{10}{12} = \dfrac{5}{6}$.
The minimum possible value of $\dfrac{p}{q}$ is when p is as small as possible (that is, 3) and q is as large as possible (that is, 21). Thus, the maximum possible value of $\dfrac{p}{q}$ is $\dfrac{3}{21} = \dfrac{1}{7}$.
The difference between these two values is $\dfrac{5}{6} - \dfrac{1}{7} = \dfrac{35}{42} - \dfrac{6}{42} = \dfrac{29}{42}$.

Answer: (A)

22. Suppose that the distance from Ginger's home to her school is d km.
Since there are 60 minutes in an hour, then $3\frac{3}{4}$ minutes (or $\frac{15}{4}$ minutes) is $\frac{15}{4} \times \frac{1}{60} = \frac{1}{16}$ of an hour.
Since Ginger walks at 4 km/h, then it takes her $\dfrac{d}{4}$ hours to walk to school.
Since Ginger runs at 6 km/h, then it takes her $\dfrac{d}{6}$ hours to run to school.
Since she saves $\frac{1}{16}$ of an hour by running, then the difference between these times is $\frac{1}{16}$ of an hour, so

$$\begin{aligned} \frac{d}{4} - \frac{d}{6} &= \frac{1}{16} \\ \frac{3d}{12} - \frac{2d}{12} &= \frac{1}{16} \\ \frac{d}{12} &= \frac{1}{16} \\ d &= \frac{12}{16} = \frac{3}{4} \end{aligned}$$

Therefore, the distance from Ginger's home to her school is $\frac{3}{4}$ km.

Answer: (E)

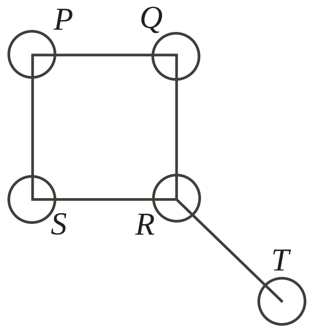
23. Suppose that the distance from line M to line L is d m.
Therefore, the total length of piece W to the left of the cut is d m.
Since piece X is 3 m from line M, then the length of piece X to the left of L is $(d-3)$ m, because 3 of the d m to the left of L are empty.
Similarly, the lengths of pieces Y and Z to the left of line L are $(d-2)$ m and $(d-1.5)$ m.
Therefore, the total length of lumber to the left of line L is

$$d + (d-3) + (d-2) + (d-1.5) = 4d - 6.5 \text{ m}$$

Since the total length of lumber on each side of the cut is equal, then this total length is $\frac{1}{2}(5+3+5+4) = 8.5$ m.
(We could instead find the lengths of lumber to the right of line L to be $5-d$, $6-d$, $7-d$, and $5.5-d$ and equate the sum of these lengths to the sum of the lengths on the left side.)
Therefore, $4d - 6.5 = 8.5$ or $4d = 15$ or $d = 3.75$, so the length of the part of piece W to the left of L is 3.75 m.

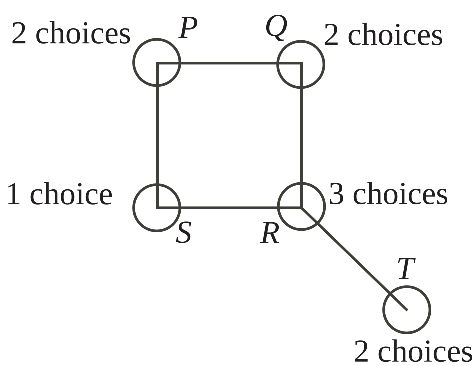
Answer: (D)

24. We label the five circles as shown in the diagram.



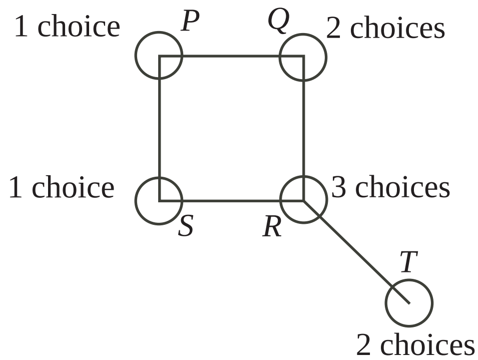
We note that there are 3 possible colours and that no two adjacent circles can be coloured the same.
Consider circle R. There are three possible colours for this circle.
For each of these colours, there are 2 possible colours for T (either of the two colours that R is not), since it cannot be the same colour as R.
Circles Q and S are then either the same colour as each other, or are different colours.

Case 1: Q and S are the same colour
In this case, there are 2 possible colours for Q (either of the colours that R is not) and 1 possibility for S (the same colour as Q).
For each of these possible colours for Q/S, there are two possible colours for P (either of the colours that Q and S are not).



In this case, there are thus $3 \times 2 \times 2 \times 1 \times 2 = 24$ possible ways of colouring the circles.

Case 2: Q and S are different colours
In this case, there are 2 possible colours for Q (either of the colours that R is not) and 1 possibility for S (since it must be different from R and different from Q).
For each of these possible colourings of Q and S, there is 1 possible colour for P (since Q and S are different colours, P is different from these, and there are only 3 colours in total).



In this case, there are thus $3 \times 2 \times 2 \times 1 \times 1 = 12$ possible ways of colouring the circles.

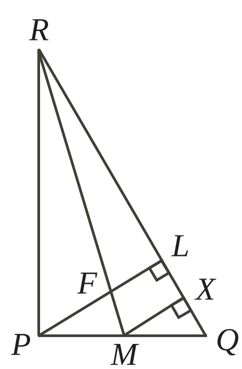
In total, there are thus $24 + 12 = 36$ possible ways to colour the circles.

Answer: (D)

25. Since $PQ = 2$ and M is the midpoint of PQ, then $PM = MQ = \frac{1}{2}(2) = 1$.
Since $\triangle PQR$ is right-angled at P, then by the Pythagorean Theorem,

$$RQ = \sqrt{PQ^2 + PR^2} = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$$

(Note that we could say that $\triangle PQR$ is a 30°-60°-90° triangle, but we do not actually need this fact.)
Since PL is an altitude, then $\angle PLR = 90^\circ$, so $\triangle RLP$ is similar to $\triangle RPQ$ (these triangles have right angles at L and P respectively, and a common angle at R).
Therefore, $\dfrac{PL}{QP} = \dfrac{RP}{RQ}$ or $PL = \dfrac{(QP)(RP)}{RQ} = \dfrac{2(2\sqrt{3})}{4} = \sqrt{3}$.
Similarly, $\dfrac{RL}{RP} = \dfrac{RP}{RQ}$ so $RL = \dfrac{(RP)(RP)}{RQ} = \dfrac{(2\sqrt{3})(2\sqrt{3})}{4} = 3$.
Therefore, $LQ = RQ - RL = 4 - 3 = 1$ and $PF = PL - FL = \sqrt{3} - FL$.
So we need to determine the length of FL.
Drop a perpendicular from M to X on RQ.



Then $\triangle MXQ$ is similar to $\triangle PLQ$, since these triangles are each right-angled and they share a common angle at Q. Since $MQ = \frac{1}{2}PQ$, then the corresponding sides of $\triangle MXQ$ are half as long as those of $\triangle PLQ$.
Therefore, $QX = \frac{1}{2}QL = \frac{1}{2}(1) = \frac{1}{2}$ and $MX = \frac{1}{2}PL = \frac{1}{2}(\sqrt{3}) = \frac{\sqrt{3}}{2}$.
Since $QX = \frac{1}{2}$, then $RX = RQ - QX = 4 - \frac{1}{2} = \frac{7}{2}$.

Now $\triangle RLF$ is similar to $\triangle RXM$ (they are each right-angled and share a common angle at R).

Therefore, $\dfrac{FL}{MX} = \dfrac{RL}{RX}$ so $FL = \dfrac{(MX)(RL)}{RX} = \dfrac{\frac{\sqrt{3}}{2}(3)}{\frac{7}{2}} = \frac{3\sqrt{3}}{7}$.

Thus, $PF = \sqrt{3} - \frac{3\sqrt{3}}{7} = \frac{4\sqrt{3}}{7}$.

Answer: (C)