

2008 Galois Contest (Grade 10)

Wednesday, April 16, 2008

- Three positive integers a , b and x form an O'Hara triple (a, b, x) if $\sqrt{a} + \sqrt{b} = x$. For example, $(1, 4, 3)$ is an O'Hara triple because $\sqrt{1} + \sqrt{4} = 3$.
 - If $(36, 25, x)$ is an O'Hara triple, determine the value of x .
 - If $(a, 9, 5)$ is an O'Hara triple, determine the value of a .
 - Determine the five O'Hara triples with $x = 6$. Explain how you found these triples.
- Determine the equation of the line passing through the points $P(0, 5)$ and $Q(6, 9)$.
 - A line, through Q , is perpendicular to PQ . Determine the equation of the line.
 - The line from (b) crosses the x -axis at R . Determine the coordinates of R .
 - Determine the area of right-angled $\triangle PQR$.
- A class of 20 students was given a two question quiz. The results are listed below:

Question number	Number of students who answered correctly
1	18
2	14

Determine the smallest possible number and the largest possible number of students that could have answered both questions correctly. Explain why these are the smallest and largest possible numbers.

- A class of 20 students was given a three question quiz. The results are listed below:

Question number	Number of students who answered correctly
1	18
2	14
3	12

Determine the smallest possible number and the largest possible number of students that could have answered all three questions correctly. Explain why these are the smallest and largest possible numbers.

- A class of 20 students was given a three question quiz. The results are listed below:

Question number	Number of students who answered correctly
1	x
2	y
3	z

where $x \geq y \geq z$ and $x + y + z \geq 40$.

Determine the smallest possible number of students who could have answered all three questions correctly in terms of x , y and z .

4. Carolyn and Paul are playing a game starting with a list of the integers 1 to n . The rules of the game are:

- Carolyn always has the first turn.
- Carolyn and Paul alternate turns.
- On each of her turns, Carolyn must remove one number from the list such that this number has at least one positive divisor other than itself remaining in the list.
- On each of his turns, Paul must remove from the list all of the positive divisors of the number that Carolyn has just removed.
- If Carolyn cannot remove any more numbers, then Paul removes the rest of the numbers.

For example, if $n = 6$, a possible sequence of moves is shown in this chart:

Player	Number(s) removed	Number(s) remaining	Notes
Carolyn	4	1, 2, 3, 5, 6	
Paul	1, 2	3, 5, 6	
Carolyn	6	3, 5	She could not remove 3 or 5
Paul	3	5	
Carolyn	None	5	Carolyn cannot remove any number
Paul	5	None	

In this example, the sum of the numbers removed by Carolyn is $4 + 6 = 10$ and the sum of the numbers removed by Paul is $1 + 2 + 3 + 5 = 11$.

- (a) Suppose that $n = 6$ and Carolyn removes the integer 2 on her first turn. Determine the sum of the numbers that Carolyn removes and the sum of the numbers that Paul removes.
- (b) If $n = 10$, determine Carolyn's maximum possible final sum. Prove that this sum is her maximum possible sum.
- (c) If $n = 14$, prove that Carolyn cannot remove 7 numbers.