2008 Fryer Contest (Grade 9)

Wednesday, April 16, 2008

1. A *magic square* is a grid of numbers in which the sum of the numbers in each row, in each column, and on each of the two main diagonals is equal to the same number (called the *magic constant*). For example,

4	3	8
9	5	1
2	7	6

is a magic square because the sum of the numbers in each row, in each column, and on each of the two main diagonals is equal to 15. (15 is the magic constant.)

   (a) A magic square is to be formed using the nine integers from 11 to 19.

      (i) Calculate the sum of the nine integers from 11 to 19.

      (ii) Determine the magic constant for this magic square and explain how you found it.

      (iii) Complete the magic square starting with the entries

18	11	
		12

.

   (b) A magic square is to be formed using the sixteen integers from 1 to 16.

      (i) Calculate the sum of the sixteen integers from 1 to 16.

      (ii) Determine the magic constant for this magic square and explain how you found it.

      (iii) Complete the magic square starting with the entries

16	3		13
5		11	
		7	12
4		14	1

.

2. If a team won 13 games and lost 7 games, its *winning percentage* was $\dfrac{13}{13+7} \times 100\% = 65\%$, because it won 13 of its 20 games.

   (a) The Sharks played 10 games and won 8 of these.
   Then they played 5 more games and won 1 of these.
   What was their final winning percentage? Show the steps that you took to find your answer.

   (b) The Emus won 4 of their first 10 games.
   The team played x more games and won all of these.
   Their final winning percentage was 70%.
   How many games did they play in total? Show the steps that you took to find your answer.

   (c) The Pink Devils started out the season with 7 wins and 3 losses.
   They lost all of their games for the rest of the season.
   Was there a point during the season when they had won exactly $\frac{2}{7}$ of their games? Explain why or why not.

3. (a) Figure 1 shows a net that can be folded to create a rectangular box. Determine the volume and the surface area of the box.

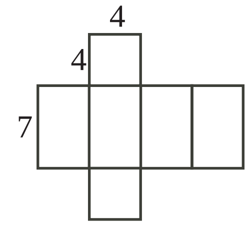

Figure 1

(b) In Figure 2, the rectangular box has dimensions 2 by 2 by 6. From point A, an ant walked to point B crossing all four of the side faces. The shortest path along which the ant could walk may be found by unfolding the box, as in Figure 3, and drawing a straight line from A to B. Determine the length of AB in Figure 3.

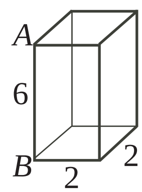

Figure 2

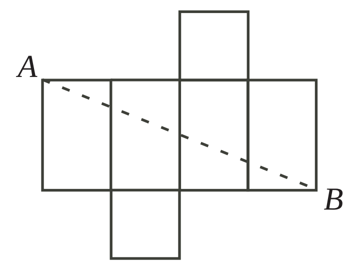

Figure 3

(c) In Figure 4, the rectangular block has dimensions 3 by 4 by 5. A caterpillar is at corner A. Determine, with justification, the shortest possible distance from A to G along the surface of the block.

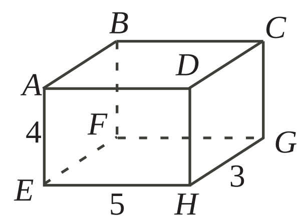

Figure 4

4. When the first 30 positive integers are written together in order, the 51-digit number

$$x = 123456789101112131415161718192021222324252627282930$$

is formed.

(a) A positive integer that is the same when read forwards or backwards is called a *palindrome*. For example, 12321 and 1221 are both palindromes.
Determine the smallest number of digits that must be removed from x so that the remaining digits can be rearranged to form a palindrome. Justify why this is the minimum number of digits.

(b) Determine the minimum number of digits that must be removed from x so that the remaining digits have a sum of 130. Justify why this is the minimum number of digits.

(c) When the first 50 positive integers are written in order, the 91-digit number

$$y = 123456789101112 \cdots 484950$$

is formed. Determine the minimum number of digits that must be removed from y so that the remaining digits have a sum of 210 and can be rearranged to form a palindrome. Justify why this is the minimum number of digits.