Canadian
Mathematics
Competition
An activity of the Centre for Education
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2008 Fermat Contest

(Grade 11)

Tuesday, February 19, 2008

Solutions

1. Calculating, $\dfrac{1^2+2^2+3^2+4^2}{1\times 2\times 3} = \dfrac{1+4+9+16}{6} = \dfrac{30}{6} = 5$.

   Answer: (D)

2. *Solution 1*
   Calculating, $6\left(\frac{3}{2}+\frac{2}{3}\right) = 6\left(\frac{3}{2}\right)+6\left(\frac{2}{3}\right) = 9+4 = 13$.

   Solution 2
   Simplifying first insides the brackets, $6\left(\frac{3}{2}+\frac{2}{3}\right) = 6\left(\frac{9}{6}+\frac{4}{6}\right) = 6\left(\frac{13}{6}\right) = 13$.

   Answer: (A)

3. Since $1+2+3+4+5+x = 21+22+23+24+25$, then

   $$x = 21-1+22-2+23-3+24-4+25-5 = 5(20) = 100$$

   Answer: (C)

4. Since an empty truck weighs 9600 kg and when the 40 crates are added the weight is 38 000 kg, then the total weight of the crates is $38\,000 - 9600 = 28\,400$ kg.
   Since there are 40 identical crates that weigh 28 400 kg, then the weight of each crate is $28\,400 \div 40 = 710$ kg.

   Answer: (E)

5. Since $\dfrac{18}{\sqrt{x}} = 2$, then $\sqrt{x} = 9$, because the number by which we must divide 18 to get 2 is 9.
   Since $\sqrt{x} = 9$, then $x = 9^2 = 81$.

   Answer: (A)

6. Since $RQ = RS$, then $\angle RSQ = \angle RQS$.
   In $\triangle QRS$, we have $\angle RQS + \angle QRS + \angle RSQ = 180^\circ$, so $2(\angle RQS) + 60^\circ = 180^\circ$.
   Thus, $\angle RQS = \frac{1}{2}(180^\circ - 60^\circ) = 60^\circ$.
   Since $PQ = PS$, then $\angle PSQ = \angle PQS$.
   In $\triangle QPS$, we have $\angle PQS + \angle QPS + \angle PSQ = 180^\circ$, so $2(\angle PQS) + 30^\circ = 180^\circ$.
   Thus, $\angle PQS = \frac{1}{2}(180^\circ - 30^\circ) = 75^\circ$.
   Therefore, $\angle PQR = \angle PQS - \angle RQS = 75^\circ - 60^\circ = 15^\circ$.

   Answer: (E)

7. *Solution 1*
   Since p is odd and q is even, then $3p$ is odd times odd (so is odd) and $2q$ is even times even (so is even).
   Therefore, $3p + 2q$ is odd plus even, which is odd.
   (Since we have found one possibility that is odd, we do not need to look at the others. We could check, though, that each of the others is always even.)

   Solution 2
   We check for a particular case of p and q, since the problem implies that the result is the same no matter what odd and even integers p and q we choose.
   We test the five choices with $p = 1$ and $q = 2$, which are odd and even, respectively.
   In this case, $2p + 3q = 8$ and $3p + 2q = 7$ and $4p + q = 6$ and $2(p + 3q) = 14$ and $pq = 2$.
   Thus, the only possibility that is odd is $3p + 2q$.

   Answer: (B)

8. *Solution 1*

The wording of the problem tells us that $a+b+c+d+e+f$ must be the same no matter what numbers abc and def are chosen that satisfy the conditions.
An example that works is $889+111=1000$.
In this case, $a+b+c+d+e+f=8+8+9+1+1+1=28$, so this must always be the value.

Solution 2

Consider performing this "long addition" by hand.
Consider first the units column.
Since $c+f$ ends in a 0, then $c+f=0$ or $c+f=10$. The value of $c+f$ cannot be 20 or more, as c and f are digits.
Since none of the digits is 0, we cannot have $c+f=0+0$ so $c+f=10$. (This means that we "carry" a 1 to the tens column.)
Since the result in the tens column is 0 and there is a 1 carried into this column, then $b+e$ ends in a 9, so we must have $b+e=9$. (Since b and e are digits, $b+e$ cannot be 19 or more.)
In the tens column, we thus have $b+e=9$ plus the carry of 1, so the resulting digit in the tens column is 0, with a 1 carried to the hundreds column.
Using a similar analysis in the hundreds column to that in the tens column, we must have $a+d=9$.
Therefore, $a+b+c+d+e+f=(a+d)+(b+e)+(c+f)=9+9+10=28$.

Answer: (D)

9. *Solution 1*

Since $\frac{1}{5}$ is equivalent to 20%, then Beshmi invests a total of $20\%+42\%=62\%$ of her savings in Companies X and Y, leaving $100\%-62\%=38\%$ for Company Z.
Since 42% of her savings is \$10 500, then 38% should be just slightly less than this amount, so of the given choices, must be \$9500.

Solution 2

Since $\frac{1}{5}$ is equivalent to 20%, then Beshmi invests a total of $20\%+42\%=62\%$ of her savings in Companies X and Y, leaving $100\%-62\%=38\%$ for Company Z.
Since 42% of her savings is \$10 500, then 1% of her savings is $\$10\,000 \div 42 = \$250$.
But $38\% = 38\times 1\%$, which is $38\times\$250=\$9500$ here.
Therefore, she invests \$9500 in Company Z.

Answer: (D)

10. The bottom left vertex of the triangle has coordinates $(0,0)$, since $y=x$ (the line with positive slope) passes through the origin.
The bottom right vertex of the triangle corresponds with the x-intercept of the line $y=-2x+3$, which we find by setting $y=0$ to obtain $-2x+3=0$ or $x=\frac{3}{2}$. Thus, the bottom right vertex is $\left(\frac{3}{2},0\right)$.
The top vertex is the point of intersection of the two lines, which we find by combining the equations of the two lines to get $x=-2x+3$ or $3x=3$ or $x=1$.
Thus, this point of intersection is $(1,1)$.
Therefore, the triangle has a base along the x-axis of length $\frac{3}{2}$ and a height of length 1 (the y-coordinate of the top vertex).
Thus, the area of the triangle is $\frac{1}{2}\left(\frac{3}{2}\right)(1)=\frac{3}{4}$.

Answer: (A)

11. Since $\dfrac{1}{x} = 2$, then $x = \frac{1}{2}$. Since $\dfrac{1}{x} = 2$ and $\dfrac{1}{x} + \dfrac{3}{y} = 3$, then $\dfrac{3}{y} = 1$, so $y = 3$.
Therefore, $x + y = \frac{1}{2} + 3 = \frac{7}{2}$.

ANSWER: (D)

12. Since Siobhan's average on the seven tests is 66, then the sum of the marks on the seven tests is $7 \times 66 = 462$.
From the given marks, $69+53+69+71+78+x+y = 462$ or $340+x+y = 462$ so $x+y = 122$.
Since the sum of x and y is constant, then for the value of x to be minimum, we need the value of y to be maximum, so $y = 100$.
Therefore, the minimum possible value of x is $122 - 100 = 22$.

ANSWER: (A)

13. Since P and Q are the centres of their respective circles, then line segment PQ passes through the point of tangency between these two circles. Therefore, the length PQ is the sum of the radii of these two circles, or $PQ = 3 + 2 = 5$.
Similarly, $PR = 3 + 1 = 4$ and $QR = 2 + 1 = 3$.
Therefore, $\triangle PQR$ has side lengths 3, 4 and 5, so is right-angled since $3^2 + 4^2 = 5^2$. In fact, the right-angle is between the sides of length 3 and 4, so the area of $\triangle PQR$ is $\frac{1}{2}(3)(4) = 6$.

ANSWER: (B)

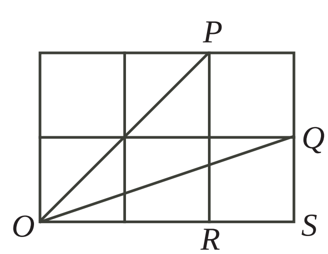
14. The circle with diameter $XZ = 12$ has radius $\frac{1}{2}(12) = 6$ so has area $\pi(6^2) = 36\pi$.
The circle with diameter $ZY = 8$ has radius $\frac{1}{2}(8) = 4$ so has area $\pi(4^2) = 16\pi$.
Thus, the total unshaded area is $36\pi + 16\pi = 52\pi$.
Since XZY is a straight line, then $XY = XZ + ZY = 12 + 8 = 20$.
The circle with diameter $XY = 20$ has radius $\frac{1}{2}(20) = 10$, so has area $\pi(10^2) = 100\pi$.
The shaded area equals the area of the circle with diameter XY minus the unshaded area, or $100\pi - 52\pi = 48\pi$.
Therefore, the ratio of the area of the shaded region to the area of the unshaded region is $48\pi : 52\pi$ or $48 : 52$ or $12 : 13$.

ANSWER: (B)

15. Since Bridget runs the second lap at $\frac{9}{10}$ of Ainslee's speed, then it takes her $\frac{10}{9}$ as long to run the lap, or $\frac{10}{9}(72) = 10(8) = 80$ seconds.
(If the lap length is d and Ainslee's speed is v, then the amount of time that Ainslee takes is $t = \dfrac{d}{v}$ and so the amount of time that Bridget takes is $\dfrac{d}{\frac{9}{10}v} = \dfrac{10}{9}\dfrac{d}{v} = \dfrac{10}{9}t$.)
Similarly, Cecilia's time for the third lap is $\frac{3}{4}(80) = 3(20) = 60$ seconds and Dana's time for the fourth lap is $\frac{5}{6}(60) = 5(10) = 50$ seconds.
Therefore, the total time is $72 + 80 + 60 + 50$ seconds or 262 seconds or 4 minutes, 22 seconds.

ANSWER: (B)

16. We add label R and S in the diagram.



Since the side length of each small square is 2, then $OR = RP = 2(2) = 4$ and $\angle ORP = 90^\circ$.
Since $\triangle ORP$ is isosceles and right-angled, then $\angle ROP = 45^\circ$.
In $\triangle OSQ$, we have $QS = 2$, $OS = 3(2) = 6$ and $\angle OSQ = 90^\circ$.
Therefore, $\tan(\angle QOS) = \frac{2}{6} = \frac{1}{3}$, so $\angle QOS \approx 18.43^\circ$.
Thus, $\angle POQ = \angle POR - \angle QOS \approx 45^\circ - 18.43^\circ = 26.57^\circ$, which, to the nearest tenth of a degree is 26.6°.

Answer: (C)

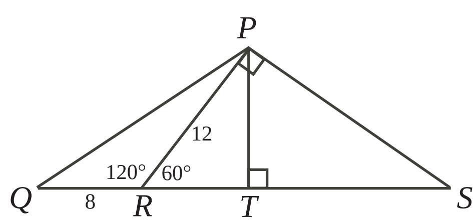
17. Suppose that these two integers are x and $x+1$, since they are consecutive.
Then $(x+1)^2 - x^2 = 199$ or $(x^2+2x+1) - x^2 = 199$ or $2x+1 = 199$ or $x = 99$.
Therefore, the two integers are 99 and 100, and the sum of their squares is $99^2 + 100^2$ or $9801 + 10\,000 = 19\,801$.

Answer: (A)

18. Since each term is obtained by adding the same number to the previous term, then the differences between pairs of consecutive terms are equal.
Looking at the first three terms, we thus have $2a - a = b - 2a$ or $b = 3a$.
Therefore, in terms of a, the first four terms are a, $2a$, $3a$, and $a - 6 - 3a = -6 - 2a$.
Since the constant difference between the terms equals a (as $2a - a = a$), then the fourth term should be $4a$, so $4a = -6 - 2a$ or $6a = -6$ or $a = -1$.
Thus, the sequence begins -1, -2, -3, -4.
The 100th term is thus -100 (which we can get by inspection or by saying that we must add the common difference 99 times to the first term, to get $-1 + 99(-1) = -100$).

Answer: (A)

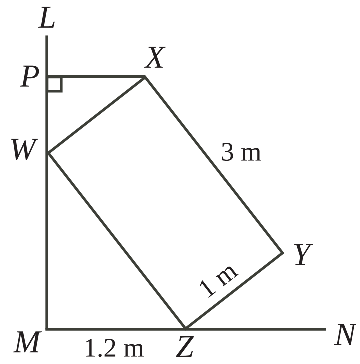
19. Since $\angle QRP = 120^\circ$ and QRS is a straight line, then $\angle PRS = 180^\circ - 120^\circ = 60^\circ$.
Since $\angle RPS = 90^\circ$, then $\triangle SRP$ is a 30°-60°-90° triangle.
Therefore, $RS = 2PR = 2(12) = 24$.
Drop a perpendicular from P to T on RS.



Since $\angle PRT = 60^\circ$ and $\angle PTR = 90^\circ$, then $\triangle PRT$ is also a 30°-60°-90° triangle.
Therefore, $PT = \frac{\sqrt{3}}{2}PR = 6\sqrt{3}$.
Consider $\triangle QPS$. We may consider QS as its base with height PT.
Thus, its area is $\frac{1}{2}(6\sqrt{3})(8+24) = 96\sqrt{3}$.

Answer: (E)

20. Draw a line from X to P on LM, so that XP is perpendicular to LM.



Since $\angle XPM = \angle PMN = 90^\circ$, then PX is parallel to MN, so the distance from X to line MN equals the length of PM.
Since $WXYZ$ is a rectangle, then $WZ = XY = 3$ m and $WX = ZY = 1$ m.
By the Pythagorean Theorem, $WM = \sqrt{WZ^2 - MZ^2} = \sqrt{3^2 - 1.2^2} = \sqrt{9 - 1.44} = \sqrt{7.56}$ m.
Since $\angle XWZ = 90^\circ$ and PWM is a straight line, then $\angle PWX + \angle XWZ + \angle ZWM = 180^\circ$, so $\angle PWX + \angle ZWM = 180^\circ - 90^\circ = 90^\circ$.
But since $\triangle XPW$ is right-angled, then

$$\angle PXW = 90^\circ - \angle PWX = 90^\circ - (90^\circ - \angle ZWM) = \angle ZWM$$

Therefore, $\triangle XPW$ is similar to $\triangle WMZ$.
This tells us that $\dfrac{PW}{MZ} = \dfrac{XW}{WZ}$, so $PW = \dfrac{MZ(XW)}{WZ} = \dfrac{1.2(1)}{3} = 0.4$ m.
Thus, $PM = PW + WM = 0.4 + \sqrt{7.56} \approx 3.1495$ m, which, to the nearest hundredth of a metre, equals 3.15 m.

Answer: (C)

21. There are 52 terms in the sum: the number 1, the number 11, and the 50 numbers starting with a 1, ending with a 1 and with 1 to 50 zeroes in between. The longest of these terms thus has 52 digits (50 zeroes and 2 ones).
When the units digits of all 52 terms are added up, their sum is 52, so the units digit of N is 2, and a 5 carried to the tens digit.
In the tens digit, there is only 1 non-zero digit: the 1 in the number 11. Therefore, using the carry, the tens digit of N is $1 + 5 = 6$.
In each of positions 3 to 52 from the right-hand end, there is only one non-zero digit, which is a 1.
Therefore, the digit in each of these positions in N is also a 1. (There is no carrying to worry about.)
Therefore, $N = 11 \cdots 1162$, where N has $52 - 2 = 50$ digits equal to 1.
This tells us that the sum of the digits of N is $50(1) + 6 + 2 = 58$.

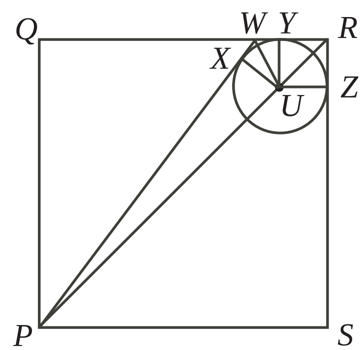
Answer: (A)

22. If the two parabolas $y = -\frac{1}{8}x^2 + 4$ and $y = x^2 - k$ do intersect, then they do so where x satisfies the equation $-\frac{1}{8}x^2 + 4 = x^2 - k$ or $\frac{9}{8}x^2 = 4 + k$.
Since $x^2 \geq 0$, then $4 + k \geq 0$, so $k \geq -4$.
(This is the condition for these two parabolae to actually intersect.)

We also want the point of intersection to be on or above the x-axis, so $y \geq 0$.
Since we know that $\frac{9}{8}x^2 = 4 + k$, then $x^2 = \frac{8}{9}(4 + k)$, so at the point(s) of intersection, $y = x^2 - k = \frac{8}{9}(4 + k) - k = \frac{32}{9} - \frac{1}{9}k$.
Since we want $y \geq 0$, then $\frac{32}{9} - \frac{1}{9}k \geq 0$, so $k \leq 32$.
Therefore, the two parabolae do intersect and intersect on or above the x-axis precisely when $-4 \leq k \leq 32$.
There are $32 - (-4) + 1 = 37$ integer values of k in this range.

Answer: (E)

23. Throughout this solution, we suppress the units (metres) until the very end. All lengths until then are given in metres.
Since square $PQRS$ has side length 4, then its diagonal PR has length $4\sqrt{2}$.
Since $PR = 4UR$, then $PU = \frac{3}{4}PR = \frac{3}{4}(4\sqrt{2}) = 3\sqrt{2}$ and $UR = \frac{1}{4}PR = \sqrt{2}$.
Suppose that the circle touches WR at Y, RS at Z, and PW at X.



Since RS is tangent to the circle at Z, then $\angle UZR = 90^\circ$.
Since $\angle PRS = 45^\circ$ (because PR is the diagonal of a square), then $\triangle UZR$ is isosceles and right-angled.
Thus, $UZ = \frac{1}{\sqrt{2}}UR = \frac{1}{\sqrt{2}}(\sqrt{2}) = 1$. That is, the radius of the circle is 1.
Therefore, $UY = UX = UZ = 1$.
Now since PW is tangent to the circle at X, then $\angle PXU = 90^\circ$.
By the Pythagorean Theorem, $PX = \sqrt{PU^2 - UX^2} = \sqrt{(3\sqrt{2})^2 - 1^2} = \sqrt{18 - 1} = \sqrt{17}$.
Also, $\sin(\angle UPX) = \dfrac{UX}{UP} = \dfrac{1}{3\sqrt{2}}$ so $\angle UPX \approx 13.63^\circ$.
Since we know the length of PX, then to determine the length of PW, we must determine the length of XW.
Since WX and WY are tangents to the circle from the same point W, then $WX = WY$, which tells us that $\triangle UWX$ and $\triangle UWY$ are congruent, so $\angle UWX = \angle UWY$.
Looking at the angles in $\triangle PWR$, we have

$$\begin{aligned} \angle WPR + \angle PWR + \angle WRP &= 180^\circ \\ 2(\angle UWX) &\approx 180^\circ - 45^\circ - 13.63^\circ \\ 2(\angle UWX) &\approx 121.37^\circ \\ \angle UWX &\approx 60.68^\circ \end{aligned}$$

In $\triangle UWX$, we have $\tan(\angle UWX) = \dfrac{UX}{XW}$ so $XW \approx \dfrac{1}{\tan(60.68^\circ)} \approx 0.5616$.
Therefore, $PW = PX + XW \approx \sqrt{17} + 0.562 \approx 4.6847$ m.
To the thousandth of a metre, this equals 4.685 m.

Answer: (C)

24. We first suppose that $a \leq b \leq c$ and consider the other cases at the end.
Since a, b and c are positive integers, then $a \geq 1$.
Can $a = 1$? If $a = 1$, then $\frac{1}{a} = 1$, so $\frac{1}{b} + \frac{1}{c} = -\frac{1}{4}$, which is not possible, since b and c are positive.
Therefore, $a > 1$.
Since $a \leq b \leq c$, then $\frac{1}{a} \geq \frac{1}{b} \geq \frac{1}{c}$, so $\frac{3}{a} = \frac{1}{a} + \frac{1}{a} + \frac{1}{a} \geq \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{3}{4}$, and so $a \leq 4$.
Thus, $a = 2$, 3 or 4.

If $a = 4$, then $\frac{1}{b} + \frac{1}{c} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$.
Since $b \leq c$, then $\frac{1}{b} \geq \frac{1}{c}$, so $\frac{1}{b} \geq \frac{1}{2}\left(\frac{1}{b} + \frac{1}{c}\right) = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$, so $b \leq 4$.
Since $a \leq b$, then $b \geq 4$, so $b = 4$.

If $a = 3$, then $\frac{1}{b} + \frac{1}{c} = \frac{3}{4} - \frac{1}{3} = \frac{5}{12}$.
Since $b \leq c$, then $\frac{1}{b} \geq \frac{1}{c}$, so $\frac{1}{b} \geq \frac{1}{2}\left(\frac{5}{12}\right) = \frac{5}{24}$, so $b \leq \frac{24}{5}$, so $b \leq 4$, since b is an integer.
Since $a \leq b$, then $b \geq 3$, so $b = 3$ or $b = 4$.

If $a = 2$, then $\frac{1}{b} + \frac{1}{c} = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$.
Since $b \leq c$, then $\frac{1}{b} \geq \frac{1}{c}$, so $\frac{1}{b} \geq \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8}$, so $b \leq 8$.
Since $\frac{1}{b} < \frac{1}{4}$ as well (because $c > 0$), then $b > 4$.
Thus, $b = 5$, 6, 7, or 8.

We now make a table of the possible values:

a	$\frac{1}{b} + \frac{1}{c}$	b	$\frac{1}{c}$	c
4	$\frac{1}{2}$	4	$\frac{1}{4}$	4
3	$\frac{5}{12}$	3	$\frac{1}{12}$	12
3	$\frac{5}{12}$	4	$\frac{1}{6}$	6
2	$\frac{1}{4}$	5	$\frac{1}{20}$	20
2	$\frac{1}{4}$	6	$\frac{1}{12}$	12
2	$\frac{1}{4}$	7	$\frac{3}{28}$	$\frac{28}{3}$
2	$\frac{1}{4}$	8	$\frac{1}{8}$	8

Thus, the triples with $a \leq b \leq c$ are $(4,4,4)$, $(3,3,12)$, $(3,4,6)$, $(2,5,20)$, $(2,6,12)$ and $(2,8,8)$.
Removing the condition $a \leq b \leq c$, we can see that any triple that solves this equation is a permutation of one of the 6 triples above, as it can be relabelled with a its smallest number, b its middle number and c its largest number.
A triple of the form (x,x,x) has only one permutation.
A triple of the form (x,x,y) (with $x \neq y$) has 3 permutations (the other two being (x,y,x) and (y,x,x)).
A triple of the form (x,y,z) (with all three different) has 6 permutations. (Try listing these out.)
Therefore, permuting the 6 possible triples above in all possible ways, the total number of

triples that solve the equation is $1+3+6+6+6+3=25$.

ANSWER: (B)

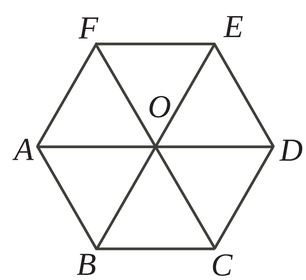
25. First, some preliminary information is needed.
Consider the base $ABCDEF$ of the sliced solid. This base is a regular hexagon. Thus, its six sides have equal length and each of its six interior angles equals 120°. (The sum of the angles of an n-gon is $(n-2)180^\circ$, which equals 720° or $6(120^\circ)$ when $n=6$.)
Let O be the centre of the hexagon.
Join each vertex to O.

Fact #1: The 6 triangles formed are equilateral

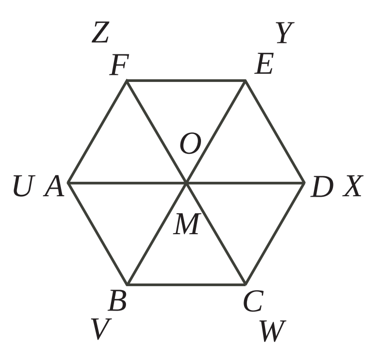
By symmetry, each of these segments bisects the angle at its vertex, creating two 60° angles. Thus, each of the six triangles formed has two 60° angles, so must have three 60° angles, so is equilateral. Therefore, the six sides and six new line segments are equal in length.

Fact #2: AOD, BOE and COF are straight lines and parallel to sides of the hexagon

Since each of the six angles at O equals 60°, then three of these angles form a straight line, so AOD, BOE and COF are straight lines, which are in addition parallel to BC and EF, CD and FA, and DE and AB, respectively. This is because of the alternate angles between pairs of lines. For example, $\angle AOF = \angle OFE = 60^\circ$, so FE and AOD are parallel.



Consider the top face $UVWXYZ$.
Let M be the point in this face directly above O.
Define $s = AU + BV + CW + DX + EY + FZ$.
Define $h(U)$ to be the height of U above A, $h(V)$ to be the height of V above B, and so on. That is, $h(U) = AU$, $h(V) = BV$, and so on.



Fact #3: $h(V) - h(U) = h(X) - h(Y)$

Note that the segments UV and YX lie directly above the segments AB and ED, and so on.
Since AB and ED are parallel and equal, then $h(V) - h(U) = h(X) - h(Y)$. This is because parallel lines in a plane have the same slope. Try visualizing a piece of paper held above a table at an angle and slice this paper with two parallel vertical planes. The lines in the paper created by these slices will have the same slope, so will have the same height change over segments of equal length.

Since AO and BC are also parallel and equal, then $h(M) - h(U) = h(W) - h(V)$. Similar equations also hold.

Fact #4: $h(M) = \frac{1}{2}(h(U) + h(X))$

We know that AO and OD are parallel and equal
Thus, $h(M) - h(U) = h(X) - h(M)$ or $h(M) = \frac{1}{2}(h(U) + h(X))$.

Similarly, $h(M) = \frac{1}{2}(h(V) + h(Y)) = \frac{1}{2}(h(W) + h(Z))$.

Fact #5: $s = 6h(M)$

Adding these last three equations,

$$3h(M) = \frac{1}{2}(h(U) + h(V) + h(W) + h(X) + h(Y) + h(Z))$$

so $s = 2(3h(M)) = 6h(M)$.

So if we can determine $h(M)$, then we can determine the sum of the lengths of the vertical segments easily.

We are now ready to solve the problem. There are a number of cases to consider. Since we can rotate the prism, it is only the relative position of the known heights that is important.

Case 1: $h(U) = 7$, $h(V) = 4$, $h(W) = 10$
Since AB and OC are parallel and equal, then $h(M) - h(W) = h(U) - h(V) = 3$, so $h(M) = 10 + 3 = 13$, so $s = 6h(M) = 6(13) = 78$.
This will turn out to be the maximum value of s.

Case 2: The heights above two opposite vertices are two of 4, 7 and 10
In this case, $h(M)$ will be the average of two of 4, 7 and 10, and so $h(M)$ is certainly less than 10, so $s = 6h(M) < 6(10) = 60$. This does not give a maximum.

Case 3: The heights 4, 7 and 10 are above consecutive vertices
To avoid duplicating Case 1, we have either $h(U) = 4$, $h(V) = 7$, $h(W) = 10$, or $h(U) = 4$, $h(V) = 10$, $h(W) = 7$.
From the analysis in Case 1, $h(M) = h(U) + h(W) - h(V)$.
In these two cases, $h(M) = 7$ or $h(M) = 1$, giving $s = 42$ or $s = 6$, neither of which is a maximum.

Case 4: None of 4, 7, 10 are adjacent
Suppose $h(U) = 4$, $h(W) = 7$ and $h(Y) = 10$. (There are no other different such configurations to consider.)
Suppose that $h(M) = x$.
Since $h(M)$ is the average of the heights above opposite vertices, then $h(V) = 2h(M) - h(Y)$ so $h(V) = 2x - 10$.
But AB and OC are parallel and equal, so $h(V) - h(U) = h(W) - h(M)$ or $2x - 10 - 4 = 7 - x$ or $3x = 21$ or $x = 7$.
Thus, $s = 6h(M) = 6x = 42$.

Having considered all possible cases, the maximum value of s (that is, the sum of the six vertical lengths) is 78.

ANSWER: (D)