



# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Fermat Contest (Grade 11)

Tuesday, February 19, 2008

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**Time:** 60 minutes

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**Calculators are permitted**

### Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site,  
<http://www.cemc.uwaterloo.ca>.

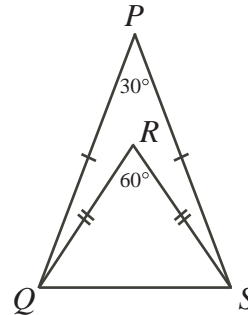
Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

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**Part A: Each correct answer is worth 5.**

- The value of  $\frac{1^2 + 2^2 + 3^2 + 4^2}{1 \times 2 \times 3}$  is  
(A) 110      (B) 22      (C)  $\frac{50}{3}$       (D) 5      (E) 14
- The value of  $6\left(\frac{3}{2} + \frac{2}{3}\right)$  is  
(A) 13      (B) 6      (C)  $\frac{13}{6}$       (D)  $\frac{29}{3}$       (E) 5
- If  $1 + 2 + 3 + 4 + 5 + x = 21 + 22 + 23 + 24 + 25$ , then the value of  $x$  is  
(A) 11      (B) 210      (C) 100      (D) 20      (E) 26
- An empty truck weighs 9600 kg. When the truck is loaded with 40 identical crates, the total weight is 38 000 kg. The weight of each crate is  
(A) 460 kg      (B) 950 kg      (C) 1190 kg      (D) 240 kg      (E) 710 kg

- If  $\frac{18}{\sqrt{x}} = 2$ , then the value of  $x$  is  
(A) 81      (B) 36      (C) 18      (D) 9      (E) 3
- In the diagram, what is the measure of  $\angle PQR$ ?  
(A)  $45^\circ$       (B)  $30^\circ$       (C)  $60^\circ$   
(D)  $75^\circ$       (E)  $15^\circ$



- If  $p$  is an odd integer and  $q$  is an even integer, which one of the following is an odd integer?  
(A)  $2p + 3q$       (B)  $3p + 2q$       (C)  $4p + q$       (D)  $2(p + 3q)$       (E)  $pq$
- Two 3-digit integers,  $abc$  and  $def$ , have the following property:

$$\begin{array}{r} a \quad b \quad c \\ + \quad d \quad e \quad f \\ \hline 1 \quad 0 \quad 0 \quad 0 \end{array}$$

None of  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$ , or  $f$  is 0. What is  $a + b + c + d + e + f$ ?

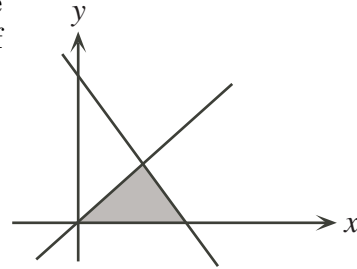
- (A) 10      (B) 19      (C) 21      (D) 28      (E) 30

9. Beshmi invested  $\frac{1}{5}$  of her savings in Company X, 42% in Company Y, and the remainder in Company Z. If Beshmi invested \$10 500 in Company Y, how much did she invest in Company Z?

(A) \$25 000    (B) \$15 500    (C) \$14 000    (D) \$9500    (E) \$5000

10. In the diagram, the shaded region is bounded by the  $x$ -axis and the lines  $y = x$ , and  $y = -2x + 3$ . The area of the shaded region is

(A)  $\frac{3}{4}$     (B)  $\frac{3}{2}$     (C)  $\frac{9}{4}$   
 (D) 1    (E)  $\frac{\sqrt{10}}{4}$



**Part B: Each correct answer is worth 6.**

11. If  $\frac{1}{x} = 2$  and  $\frac{1}{x} + \frac{3}{y} = 3$ , then the value of  $x + y$  is

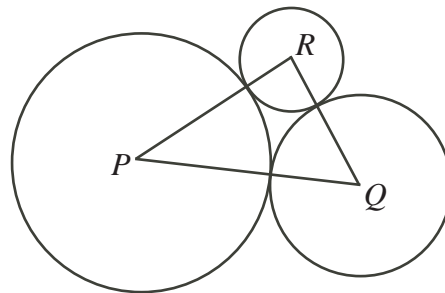
(A) 3    (B)  $\frac{5}{6}$     (C)  $\frac{7}{3}$     (D)  $\frac{7}{2}$     (E)  $\frac{4}{3}$

12. On seven tests, each out of 100 marks, Siobhan received marks of 69, 53, 69, 71, 78,  $x$ , and  $y$ . If her average mark on the seven tests is 66, then the minimum possible value of  $x$  is

(A) 22    (B) 68    (C) 61    (D) 53    (E) 0

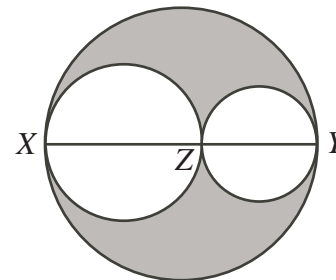
13. In the diagram, the circles with centres  $P$ ,  $Q$  and  $R$  have radii 3, 2 and 1 respectively. Each circle touches the other two as shown. The area of  $\triangle PQR$  is

(A) 12    (B) 6    (C) 7.5  
 (D) 10    (E) 4



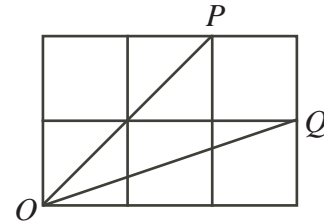
14. In the diagram,  $Z$  lies on  $XY$  and the three circles have diameters  $XZ$ ,  $ZY$  and  $XY$ . If  $XZ = 12$  and  $ZY = 8$ , then the ratio of the area of the shaded region to the area of the unshaded region is

(A) 12 : 25    (B) 12 : 13    (C) 1 : 1  
 (D) 1 : 2    (E) 2 : 3



15. In a relay race, Ainslee runs the first lap in 72 seconds. Bridget runs the next lap at  $\frac{9}{10}$  of Ainslee's speed. Cecilia runs the next lap at  $\frac{4}{3}$  of Bridget's speed. Dana runs the last lap at  $\frac{6}{5}$  of Cecilia's speed. What is their total time, to the nearest second?
- (A) 4 minutes, 48 seconds  
 (B) 4 minutes, 22 seconds  
 (C) 5 minutes, 27 seconds  
 (D) 4 minutes, 37 seconds  
 (E) 3 minutes, 46 seconds

16. In the diagram, the six small squares all have side length 2. Lines are drawn from  $O$  to  $P$  and  $O$  to  $Q$ . The measure of  $\angle POQ$  in degrees, accurate to one decimal place, is



- (A) 15.0      (B) 25.5      (C) 26.6  
 (D) 22.5      (E) 30.0

17. The difference between the squares of two consecutive integers is 199. The sum of the squares of these two consecutive integers is

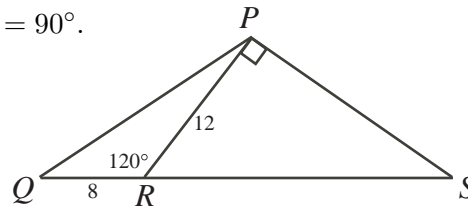
- (A) 19 801      (B) 39 601      (C) 19 602      (D) 20 201      (E) 19 405

18. An *arithmetic sequence* is a sequence in which each term after the first is obtained by adding a constant to the previous term.

If the first four terms of an arithmetic sequence are  $a$ ,  $2a$ ,  $b$ , and  $a - 6 - b$  for some numbers  $a$  and  $b$ , then the value of the 100th term is

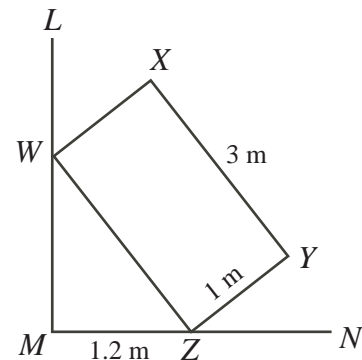
- (A)  $-100$       (B)  $-300$       (C)  $150$       (D)  $-150$       (E)  $100$

19. In the diagram,  $R$  is on  $QS$  and  $QR = 8$ . Also,  $PR = 12$ ,  $\angle PRQ = 120^\circ$ , and  $\angle RPS = 90^\circ$ . What is the area of  $\triangle QPS$ ?



- (A)  $72\sqrt{3}$       (B) 72      (C) 36  
 (D)  $60\sqrt{3}$       (E)  $96\sqrt{3}$

20. In the diagram,  $LM$  is perpendicular to  $MN$ . Rectangle  $WXYZ$  has  $W$  on  $LM$  and  $Z$  on  $MN$ . Also,  $YZ = 1$  m,  $XY = 3$  m and  $MZ = 1.2$  m. What is the distance from  $X$  to line  $MN$ , to the nearest hundredth of a metre?

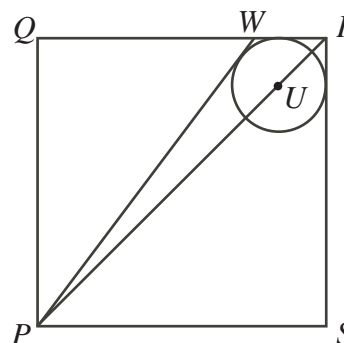


- (A) 2.75 m      (B) 3.67 m      (C) 3.15 m  
 (D) 3.26 m      (E) 3.63 m

**Part C: Each correct answer is worth 8.**

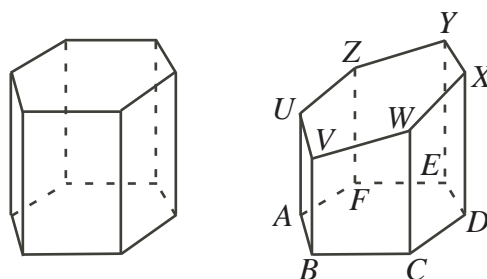
21. Suppose  $N = 1 + 11 + 101 + 1001 + 10001 + \dots + \overbrace{1000\dots00001}^{50 \text{ zeroes}}$ .  
 When  $N$  is calculated and written as a single integer, the sum of its digits is  
 (A) 58      (B) 99      (C) 55      (D) 50      (E) 103
22. For how many integers  $k$  do the parabolas with equations  $y = -\frac{1}{8}x^2 + 4$  and  $y = x^2 - k$  intersect on or above the  $x$ -axis?  
 (A) 9      (B) 32      (C) 33      (D) 36      (E) 37

23. Square  $PQRS$  has side length 4 m. Point  $U$  is on  $PR$  with  $PR = 4UR$ . A circle centered at  $U$  touches two sides of the square.  $PW$  is a tangent to the circle, with  $W$  on  $QR$ . The length of  $PW$ , to the nearest thousandth of a metre, is  
 (A) 4.123 m      (B) 4.472 m      (C) 4.685 m  
 (D) 4.726 m      (E) 4.767 m



24. The number of triples  $(a, b, c)$  of positive integers such that  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{3}{4}$  is  
 (A) 16      (B) 25      (C) 31      (D) 19      (E) 34

25. A right regular hexagonal prism is sliced as shown in the diagram. The bottom of the new solid is a regular hexagon  $ABCDEF$ . The six side faces are trapezoids perpendicular to  $ABCDEF$ . The top is a hexagon  $UVWXYZ$  that is not necessarily a regular hexagon.



Of the six edges  $AU$ ,  $BV$ ,  $CW$ ,  $DX$ ,  $EY$ , and  $FZ$ , three have lengths 4, 7 and 10. The largest possible value for  $AU + BV + CW + DX + EY + FZ$  is

- (A) 42      (B) 51      (C) 69      (D) 78      (E) 91



## Canadian Mathematics Competition



### *For students...*

Thank you for writing the 2008 Fermat Contest!  
In 2007, more than 86 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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