2007 Galois Contest (Grade 10)
Wednesday, April 18, 2007

1. Jim shops at a strange fruit store. Instead of putting prices on each item, the mathematical store owner will answer questions about combinations of items.

   (a) In Aisle 1, Jim receives the following answers to his questions:

Jim's Question	Answer
What is the sum of the prices of an Apple and a Cherry?	62 cents
What is the sum of the prices of a Banana and a Cherry?	66 cents

   What is difference between the prices of an Apple and a Banana? Which has a higher price? Explain how you obtained your answer.

   (b) In Aisle 2, Jim receives the following answers to his questions:

Jim's Question	Answer
What is the sum of the prices of a Mango and a Nectarine?	60 cents
What is the sum of the prices of a Pear and a Nectarine?	60 cents
What is the sum of the prices of a Mango and a Pear?	68 cents

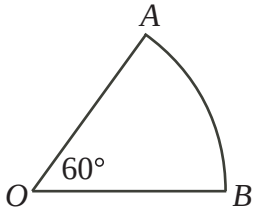
   What is the price of a Pear? Explain how you obtained your answer.

   (c) In Aisle 3, Jim receives the following answers to his questions:

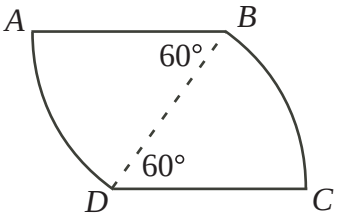
Jim's Question	Answer
What is the sum of the prices of a Tangerine and a Lemon?	60 cents
How much more does a Tangerine cost than a Grapefruit?	6 cents
What is the sum of the prices of Grapefruit, a Tangerine and a Lemon?	94 cents

   What is the price of a Lemon? Explain how you obtained your answer.

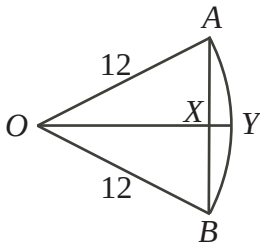
2. (a) In the diagram, what is the perimeter of the sector of the circle with radius 12? Explain how you obtained your answer.



   (b) Two sectors of a circle of radius 12 are placed side by side, as shown. Determine the *area* of figure $ABCD$. Explain how you obtained your answer.

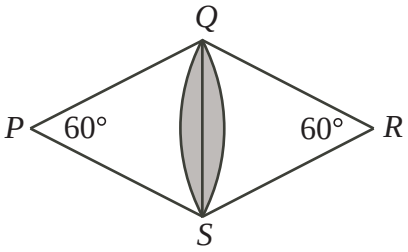


   (c) In the diagram, AOB is a sector of a circle with $\angle AOB = 60°$. OY is drawn perpendicular to AB and intersects AB at X. What is the length of XY? Explain how you obtained your answer.

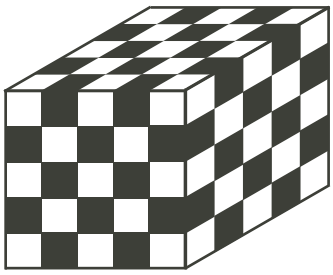


   (d) See over...

(d) Two sectors of a circle of radius 12 overlap as shown. Determine the area of the shaded region. Explain how you obtained your answer.



3. (a) Each face of a 5 by 5 by 5 wooden cube is divided into 1 by 1 squares. Each square is painted black or white, as shown. Next, the cube is cut into 1 by 1 by 1 cubes. How many of these cubes have *at least* two painted faces? Explain how you obtained your answer.



(b) A $(2k+1)$ by $(2k+1)$ by $(2k+1)$ cube, where k is a positive integer, is painted in the same manner as the 5 by 5 by 5 cube with white squares in the corners. Again, the cube is cut into 1 by 1 by 1 cubes.

i. In terms of k, how many of these cubes have *exactly* two white faces? Explain how you obtained your answer.

ii. Prove that there is no value of k for which the number of cubes having *at least* two white faces is 2006.

4. Jill has a container of small cylindrical rods in six different colours. Each colour of rod has a different length as summarized in the chart.

Colour	Length
Green	3 cm
Pink	4 cm
Yellow	5 cm
Black	7 cm
Violet	8 cm
Red	9 cm

These rods can be attached together to form a pole.
There are 2 ways to choose a set of yellow and green rods that will form a pole 29 cm in length: 8 green rods and 1 yellow rod OR 3 green rods and 4 yellow rods.

(a) How many different sets of yellow and green rods can be chosen that will form a pole 62 cm long? Explain how you obtained your answer.

(b) Among the green, yellow, black and red rods, find, with justification, two colours for which it is impossible to make a pole 62 cm in length using only rods of those two colours.

(c) If at least 81 rods of each of the colours green, pink, violet, and red must be used, how many different sets of rods of these four colours can be chosen that will form a pole 2007 cm in length? Explain how you got your answer.