Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Tuesday, February 20, 2007

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $8 + 2(3^2)$ is

   (A) 26 **(B)** 90 **(C)** 41 **(D)** 44 **(E)** 60

2. The value of $\dfrac{7+21}{14+42}$ is

   (A) $\frac{1}{3}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** 1

3. If $3x - 2x + x = 3 - 2 + 1$, then x equals

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. The table shows the pay Leona earned for two different shifts at the same fixed hourly rate. How much will she earn for a five hour shift at this rate?

Shift	Total Pay
3 hours	\$24.75
6 hours	\$49.50

   (A) \$43.75 **(B)** \$46.25 **(C)** \$38.75
   (D) \$36.25 **(E)** \$41.25

5. $\frac{1}{4}$ of 100 is equal to

   (A) 20% of 200 **(B)** 10% of 250 **(C)** 15% of 100 **(D)** 25% of 50 **(E)** 5% of 300

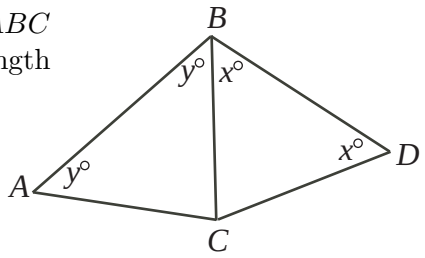
6. If $a = 2$ and $b = 5$, which of the following expressions has the greatest value?

   (A) $\dfrac{a}{b}$ **(B)** $\dfrac{b}{a}$ **(C)** $a - b$ **(D)** $b - a$ **(E)** $\frac{1}{2}a$

7. The mean (average) of 6, 9 and 18 is equal to the mean (average) of 12 and y. What is the value of y?

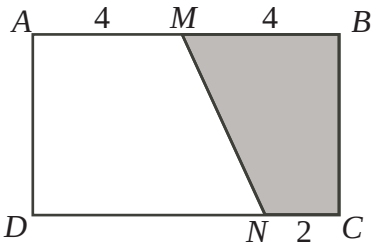
   (A) 22 **(B)** 21 **(C)** 10 **(D)** 11 **(E)** 5

8. In the diagram, triangles ABC and CBD are isosceles. The perimeter of $\triangle CBD$ is 19, the perimeter of $\triangle ABC$ is 20, and the length of BD is 7. What is the length of AB?



   (A) 5 **(B)** 6 **(C)** 7
   (D) 8 **(E)** 9

9. In the diagram, the area of rectangle $ABCD$ is 40. The area of $MBCN$ is



   (A) 15 **(B)** 10 **(C)** 30
   (D) 12 **(E)** 16

10. The first term in a sequence is x. Each of the following terms is obtained by doubling the previous term and then adding 4. If the third term is 52, then x equals

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Part B: Each correct answer is worth 6.

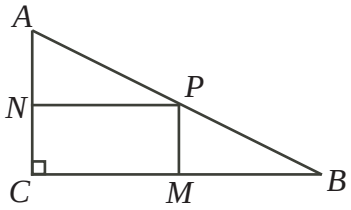
11. Ivan trained for a cross-country meet.
On Monday, he ran a certain distance.
On Tuesday, he ran twice as far as he ran on Monday.
On Wednesday, he ran half as far as he ran on Tuesday.
On Thursday, he ran half as far as he ran on Wednesday.
On Friday, he ran twice as far as he ran on Thursday.
If the shortest distance that he ran on any of the five days is 5 km, how far did he run in total?

(A) 55 km **(B)** 25 km **(C)** 27.5 km **(D)** 17.5 km **(E)** 50 km

12. The point $(0, 0)$ is reflected in the vertical line $x = 1$. When its image is then reflected in the line $y = 2$, the resulting point is

(A) $(0, 0)$ **(B)** $(2, 0)$ **(C)** $(4, 4)$ **(D)** $(2, 2)$ **(E)** $(2, 4)$

13. In the diagram, $\triangle ABC$ is right-angled at C. Also, points M, N and P are the midpoints of sides BC, AC and AB, respectively. If the area of $\triangle APN$ is 2 cm^2, then the area of $\triangle ABC$ is



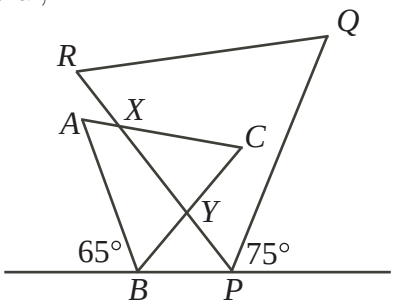
(A) 8 cm^2 **(B)** 16 cm^2 **(C)** 6 cm^2
(D) 4 cm^2 **(E)** 12 cm^2

14. If $\dfrac{3}{x-3} + \dfrac{5}{2x-6} = \dfrac{11}{2}$, then the value of $2x - 6$ is

(A) 2 **(B)** 12 **(C)** 6 **(D)** 8 **(E)** 10

15. In the diagram, if $\triangle ABC$ and $\triangle PQR$ are equilateral, then $\angle CXY$ equals

(A) 30° **(B)** 35° **(C)** 40°
(D) 45° **(E)** 50°



16. At Springfield University, there are 10 000 students, and there are as many male students as female students. Each student is enrolled either in the Arts program or Science program (but not in both); 60% of the students are in the Arts program. Also, 40% of the Science students are male. To the nearest percent, what percentage of the Arts students are female?

(A) 50% **(B)** 52% **(C)** 26% **(D)** 65% **(E)** 43%

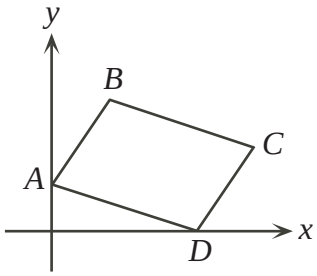
17. On an island there are two types of inhabitants: Heroes who always tell the truth and Villains who always lie. Four inhabitants are seated around a table. When each is asked "Are you a Hero or a Villain?", all four reply "Hero". When asked "Is the person on your right a Hero or a Villain?", all four reply "Villain". How many Heroes are present?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

18. There are a certain number of red balls, green balls and blue balls in a bag. Of the balls in the bag, $\frac{1}{3}$ are red and $\frac{2}{7}$ are blue. The number of green balls in the bag is 8 less than twice the number of blue balls. The number of green balls in the bag is

(A) 12 **(B)** 16 **(C)** 20 **(D)** 24 **(E)** 28

19. In the diagram, the four points have coordinates $A(0,1)$, $B(1,3)$, $C(5,2)$, and $D(4,0)$. What is the area of quadrilateral $ABCD$?



(A) 9 **(B)** 3 **(C)** 6
(D) $\sqrt{85}$ **(E)** $2\sqrt{5}+2\sqrt{17}$

20. What is the largest integer n for which $3(n^{2007}) < 3^{4015}$?

(A) 2 **(B)** 3 **(C)** 6 **(D)** 8 **(E)** 9

Part C: Each correct answer is worth 8.

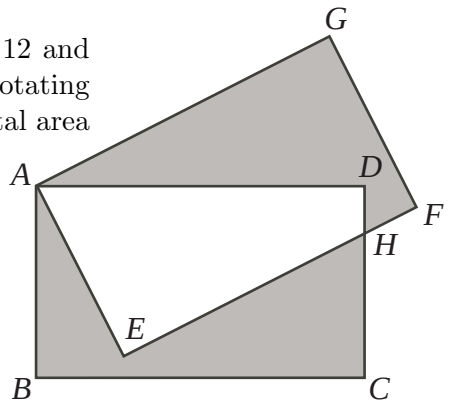
21. In a soccer league with 6 teams (P, Q, R, S, T, W), each team must eventually play each other team exactly once. So far, P has played one match, Q has played 2 matches, R has played 3 matches, S has played 4 matches, and T has played 5 matches. How many matches has W played so far?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

22. Five positive integers are listed in increasing order. The difference between any two consecutive numbers in the list is three. The fifth number is a multiple of the first number. How many different such lists of five integers are there?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

23. In the diagram, $ABCD$ is rectangle with $AB = 12$ and $BC = 18$. Rectangle $AEFG$ is formed by rotating $ABCD$ about A through an angle of 30°. The total area of the shaded regions is closest to

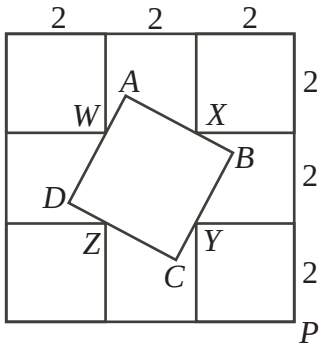


(A) 202.8 **(B)** 203.1 **(C)** 203.4
(D) 203.7 **(E)** 204.0

24. The number 8 is the sum and product of the numbers in the collection of four positive integers $\{1, 1, 2, 4\}$, since $1 + 1 + 2 + 4 = 8$ and $1 \times 1 \times 2 \times 4 = 8$. The number 2007 can be made up from a collection of n positive integers that multiply to 2007 and add to 2007. What is the smallest value of n with $n > 1$?

(A) 1171 **(B)** 1337 **(C)** 1551 **(D)** 1777 **(E)** 1781

25. In the diagram, four squares of side length 2 are placed in the corners of a square of side length 6. Each of the points W, X, Y, and Z is a vertex of one of the small squares. Square $ABCD$ can be constructed with sides passing through W, X, Y, and Z. The maximum possible distance from A to P is closest to



(A) 5.2 **(B)** 5.4 **(C)** 5.6

(D) 5.8 **(E)** 6.0

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Thank you for writing the 2007 Cayley Contest!
In 2006, more than 90 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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