Canadian
Mathematics
Competition
An activity of the Centre for Education
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Grade 8 solutions follow the Grade 7 solutions

2006 Gauss Contests

(Grades 7 and 8)

Wednesday, May 10, 2006

Solutions

Canadian Mathematics Competition Faculty and Staff

Gauss Contest Committee

Grade 7

Grade 8 solutions follow the Grade 7 solutions

1. Calculating, $(8 \times 4) + 3 = 32 + 3 = 35$.

   ANSWER: (D)

2. Since the angles along a straight line add to 180°, then $x^\circ + 40^\circ = 180^\circ$ or $x + 40 = 180$ or $x = 140$.

   ANSWER: (B)

3. To determine the number of \$50 bills, we divide the total amount of money by 50, to get $10\,000 \div 50 = 200$ bills.
   Therefore, Mikhail has 200 \$50 bills.

   ANSWER: (B)

4. The figure has 8 sides, each of equal length.
   Since the length of each side is 2, then the perimeter of the figure is $8 \times 2 = 16$.

   ANSWER: (A)

5. Using a common denominator, $\frac{2}{5} + \frac{1}{3} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$.

   ANSWER: (C)

6. Calculating by determining each product first,
   $$6 \times 100\,000 + 8 \times 1000 + 6 \times 100 + 7 \times 1 = 600\,000 + 8000 + 600 + 7 = 608\,607$$

   ANSWER: (C)

7. Since $3 + 5x = 28$, then $5x = 28 - 3 = 25$ so $x = \dfrac{25}{5} = 5$.

   ANSWER: (C)

8. Calculating, $9^2 - \sqrt{9} = 9 \times 9 - \sqrt{9} = 81 - 3 = 78$.

   ANSWER: (E)

9. In total, there are $2 + 5 + 4 = 11$ balls in the bag.
   Since there are 5 yellow balls, then the probability of choosing a yellow ball is $\frac{5}{11}$.

   ANSWER: (B)

10. Since the left edge of the block is at the "3" on the ruler and the right edge of the block is between the "5" and "6", then the length of the block is between 2 and 3.
    Looking at the possible choices, the only choice between 2 and 3 is (C) or 2.4 cm.
    (Looking again at the figure, the block appears to end roughly halfway between the "5" and the "6", so 2.4 cm is reasonable.)

    ANSWER: (C)

11. *Solution 1*
    Since the sales tax is 15%, then the total price for the CD including tax is $1.15 \times \$14.99 = \$17.2385$ which rounds to \$17.24.

    Solution 2
    Since the sales tax is 15%, then the amount of tax on the CD which costs \$14.99 is $0.15 \times \$14.99 = \$2.2485$, which rounds to \$2.25.
    Therefore, the total price of the CD including tax is $\$14.99 + \$2.25 = \$17.24$.

    ANSWER: (A)

12. *Solution 1*
Since the pool has dimensions 6 m by 12 m by 4 m, then its total volume is $6 \times 12 \times 4 = 288$ m^3. Since the pool is only half full of water, then the volume of water in the pool is $\frac{1}{2} \times 288$ m^3 or 144 m^3.

Solution 2
Since the pool is half full of water, then the depth of water in the pool is $\frac{1}{2} \times 4 = 2$ m.
Therefore, the portion of the pool which is filled with water has dimensions 6 m by 12 m by 2 m, and so has volume $6 \times 12 \times 2 = 144$ m^3.

Answer: (E)

13. To determine the number that must be added 8 to give the result -5, we subtract 8 from -5 to get $(-5) - 8 = -13$. Checking, $8 + (-13) = -5$.

Answer: (D)

14. *Solution 1*
Since AOB is a diameter of the circle, then $\angle AOB = 180^\circ$.
We are told that the angle in the "Winter" sector is a right angle (or 90°). Also, we are told that the angle in the "Spring" sector is 60°.
Therefore, the angle in the "Fall" sector is $180^\circ - 90^\circ - 60^\circ = 30^\circ$.
What fraction of the complete circle is 30°?
Since the whole circle has 360°, then the fraction is $\dfrac{30^\circ}{360^\circ} = \dfrac{1}{12}$.
Therefore, $\frac{1}{12}$ of the students chose fall as their favourite season, or $\frac{1}{12} \times 600 = 50$ students in total.

Solution 2
Since AOB is a diameter of the circle, then $\frac{1}{2}$ of the students chose summer as their favourite season, or $\frac{1}{2} \times 600 = 300$ students in total.
Since the angle in the "Winter" sector is a right angle (or 90°), then $\frac{1}{4}$ of the students (since 4 right angles make up a complete circle) chose Winter as their favourite season, or $\frac{1}{4} \times 600 = 150$ students in total.
Since the angle in the "Spring" sector is 60°, then $\dfrac{60^\circ}{360^\circ} = \dfrac{1}{6}$ of the students chose Spring as their favourite season, or $\frac{1}{6} \times 600 = 100$ students in total.
Since there were 600 students in total, then the number who chose Fall as their favourite season was $600 - 300 - 150 - 100 = 50$.

Answer: (B)

15. Since Harry charges 50% more for each additional hour as he did for the previous hour, then he charges 1.5 or $\frac{3}{2}$ times as much as he did for the previous hour.
Harry charges \$4 for the first hour.
Harry then charges $\frac{3}{2} \times \$4 = \$6$ for the second hour.
Harry then charges $\frac{3}{2} \times \$6 = \$9$ for the third hour.
Harry then charges $\frac{3}{2} \times \$9 = \$\frac{27}{2} = \$13.50$ for the fourth hour.
Therefore, for 4 hours of babysitting, Harry would earn $\$4 + \$6 + \$9 + \$13.50 = \$32.50$.

Answer: (C)

16. *Solution 1*

We obtain fractions equivalent to $\frac{5}{8}$ by multiplying the numerator and denominator by the same number.
The sum of the numerator and denominator of $\frac{5}{8}$ is 13, so when we multiply the numerator and denominator by the same number, the sum of the numerator and denominator is also multiplied by this same number.
Since $91 = 13 \times 7$, then we should multiply the numerator and denominator both by 7 to get a fraction $\frac{5\times7}{8\times7} = \frac{35}{56}$ equivalent to $\frac{5}{8}$ whose numerator and denominator add up to 91.
The difference between the denominator and numerator in this fraction is $56 - 35 = 21$.

Solution 2

We make a list of the fractions equivalent to $\frac{5}{8}$ by multiplying the numerator and denominator by the same number, namely 2, 3, 4, and so on:

$$\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \frac{25}{40}, \frac{30}{48}, \frac{35}{56}, \ldots$$

Since the numerator and denominator of $\frac{35}{56}$ add to 91 (since $35 + 56 = 91$), then this is the fraction for which we are looking.
The difference between the denominator and numerator is $56 - 35 = 21$.

Solution 3

We obtain fractions equivalent to $\frac{5}{8}$ by multiplying the numerator and denominator by the same number.
If this number is n, then a fraction equivalent to $\frac{5}{8}$ is $\frac{5n}{8n}$.
For the numerator and denominator to add up to 91, we must have $5n + 8n = 91$ or $13n = 91$ or $n = 7$.
Therefore, the fraction for which we are looking is $\frac{5\times7}{8\times7} = \frac{35}{56}$.
The difference between the denominator and numerator is $56 - 35 = 21$.

Answer: (A)

17. *Solution 1*

Since the shoe is 28 cm long and fits 15 times along one edge of the carpet, then one dimension of the carpet is $15 \times 28 = 420$ cm.
Since the shoe fits 10 times along another edge of the carpet, then one dimension of the carpet is $10 \times 28 = 280$ cm.
Therefore, then area of the carpet is $420 \times 280 = 117\,600$ cm^2.

Solution 2

Since the shoe fits along one edge of the carpet 15 times and along another edge 10 times, then the area of the carpet is $15 \times 10 = 150$ square shoes.
Since the length of the shoe is 28 cm, then

$$1 \text{ square shoe} = 1 \text{ shoe} \times 1 \text{ shoe} = 28 \text{ cm} \times 28 \text{ cm} = 784 \text{ cm}^2$$

Therefore, the area of the carpet in cm^2 is $150 \times 784 = 117\,600$ cm^2.

Answer: (E)

18. *Solution 1*

Since Keiko takes 120 seconds to run 3 times around the track, then it takes her $\frac{1}{3} \times 120 = 40$ seconds to run 1 time around the track.

Since Leah takes 160 seconds to run 5 times around the track, then it takes her $\frac{1}{5} \times 160 = 32$ seconds to run 1 time around the track.
Since Leah takes less time to run around the track than Keiko, then she is the faster runner.
Since Leah takes 32 seconds to run the 150 m around the track, then her speed is $\frac{150 \text{ m}}{32 \text{ s}} = 4.6875 \text{ m/s} \approx 4.69 \text{ m/s}$.
Therefore, Leah is the faster runner and her speed is approximately 4.69 m/s.

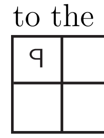
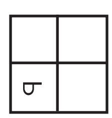
Solution 2
In 120 seconds, Keiko runs 3 times around the track, or $3 \times 150 = 450$ m in total. Therefore, her speed is $\frac{450 \text{ m}}{120 \text{ s}} = 3.75 \text{ m/s}$.
In 160 seconds, Leah runs 5 times around the track, or $5 \times 150 = 750$ m in total. Therefore, her speed is $\frac{750 \text{ m}}{160 \text{ s}} = 4.6875 \text{ m/s} \approx 4.69 \text{ m/s}$.
Since Leah's speed is larger, she is the faster runner and her speed is approximately 4.69 m/s.

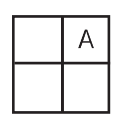
Answer: (D)

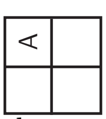
19. *Solution 1*
In one minute, there are 60 seconds.
In one hour, there are 60 minutes, so there are $60 \times 60 = 3600$ seconds.
In one day, there are 24 hours, so there are $24 \times 3600 = 86\,400$ seconds.
Therefore, 10^6 seconds is equal to $\frac{10^6}{86\,400} \approx 11.574$ days, which of the given choices is closest to 10 days.

Solution 2
Since there are 60 seconds in one minute, then 10^6 seconds is $\frac{10^6}{60} \approx 16\,666.67$ minutes.
Since there are 60 minutes in one hour, then 16 666.67 minutes is $\frac{16\,666.67}{60} \approx 277.78$ hours.
Since there are 24 hours in one day, then 277.78 hours is $\frac{277.78}{24} \approx 11.574$ days, which of the given choices is closest to 10 days.

Answer: (B)

20. One possible way to transform the initial position of the "P" to the final position of the "P" is to reflect the grid in the vertical line in the middle to obtain  and then rotate the grid 90° counterclockwise about the centre to obtain  .

Applying these transformations to the grid containing the "A", we obtain 

and then  . (There are many other possible combinations of transformations which will produce the same resulting image with the "P"; each of these combinations will produce the same result with the "A".)

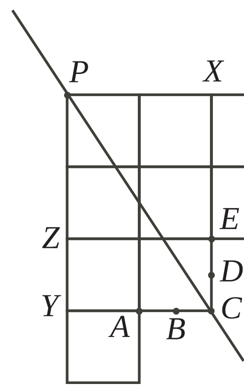
Answer: (B)

21. *Solution 1*
Between x a.m. and x p.m. there are 12 hours. (For example, between 10 a.m. and 10 p.m. there are 12 hours.)
Therefore, Gail works for 12 hours on Saturday.

Solution 2
From x a.m. until 12 noon, the number of hours which Gail works is $12 - x$.
From 12 noon until x p.m., she works x hours.
Thus, the total number of hours that Gail works is $(12 - x) + x = 12$.

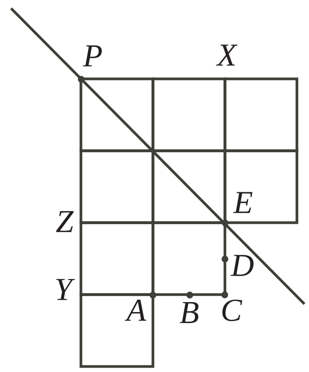
Answer: (E)

22. As an initial guess, let us see what happens when the line passes through C.



Since each square is a unit square, then the area of rectangle $PXCY$ is $2 \times 3 = 6$, and so the line through P and C cuts this area in half, leaving 3 square units in the bottom piece and 3 square units in the top piece.
(A fact has been used here that will be used several times in this solution: If a line passes through two diagonally opposite vertices of a rectangle, then it cuts the rectangle into two pieces of equal area (since it cuts the rectangle into two congruent triangles).)
In the bottom piece, we have not accounted for the very bottom unit square, so the total area of the bottom piece is 4 square units.
In the top piece, we have not accounted for the two rightmost unit squares, so the total area of the top piece is 5 square units.
So putting the line through C does not produce two pieces of equal area, so C is not the correct answer.
Also, since the area of the bottom piece is larger than the area of the top piece when the line passes through C, we must move the line up to make the areas equal (so neither A nor B can be the answer).

Should the line pass through E?

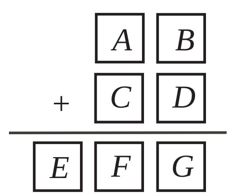


If so, then the line splits the square $PXEZ$ (of area 4) into two pieces of area 2.
Then accounting for the remaining squares, the area of the bottom piece is $2+3=5$ and the area of the top piece is $2+2=4$.
So putting the line through E does not produce two pieces of equal area, so E is not the correct answer.
By elimination, the correct answer should be D.

(We should verify that putting the line through D does indeed split the area in half.
The total area of the shape is 9, since it is made up of 9 unit squares.
If the line goes through D, the top piece consists of $\triangle PXD$ and 2 unit squares.
The area of $\triangle PXD$ is $\frac{1}{2}\times 2\times\frac{5}{2}=\frac{5}{2}$, since $PX=2$ and $XD=\frac{5}{2}$.
Thus, the area of the top piece is $\frac{5}{2}+2=\frac{9}{2}$, which is exactly half of the total area, as required.)

Answer: (D)

23. We label the blank spaces to make them easier to refer to.



Since we are adding two 2-digit numbers, then their sum cannot be 200 or greater, so E must be a 1 if the sum is to have 3 digits.

Where can the digit 0 go?
Since no number can begin with a 0, then neither A nor C can be 0.
Since each digit is different, then neither B and D can be 0, otherwise both D and G or B and G would be the same.
Therefore, only F or G could be 0.

Since we are adding two 2-digit numbers and getting a number which is at least 100, then $A+C$ must be at least 9. (It could be 9 if there was a "carry" from the sum of the units digits.)
This tells us that A and C must be 3 and 6, 4 and 5, 4 and 6, or 5 and 6.

If G was 0, then B and D would have to 4 and 6 in some order. But then the largest that A and C could be would be 3 and 5, which are not among the possibilities above.
Therefore, G is not 0, so $F=0$.

$$\begin{array}{r} \boxed{A}\,\boxed{B} \\ +\ \boxed{C}\,\boxed{D} \\ \hline \boxed{1}\,\boxed{0}\,\boxed{G} \end{array}$$

So the sum of A and C is either 9 or 10, so A and C are 3 and 6, 4 and 5, or 4 and 6.
In any of these cases, the remaining possibilities for B and D are too small to give a carry from the units column to the tens column.
So in fact, A and C must add to 10, so A and C are 4 and 6 in some order.
Let's try $A = 4$ and $C = 6$.

$$\begin{array}{r} \boxed{4}\,\boxed{B} \\ +\ \boxed{6}\,\boxed{D} \\ \hline \boxed{1}\,\boxed{0}\,\boxed{G} \end{array}$$

The remaining digits are 2, 3 and 5. To make the addition work, B and D must be 2 and 3 and G must be 5. (We can check that either order for B and D works, and that switching the 4 and 6 will also work.)

So the units digit of the sum must be 5, as in the example

$$\begin{array}{r} \boxed{4}\,\boxed{2} \\ +\ \boxed{6}\,\boxed{3} \\ \hline \boxed{1}\,\boxed{0}\,\boxed{5} \end{array}$$

(Note that we could have come up with this answer by trial and error instead of this logical procedure.)

Answer: (D)

24. The sum of any two sides of a triangle must be bigger than the third side.
(When two sides are known to be equal, we only need to check if the sum of the two equal sides is longer than the third side, since the sum of one of the equal sides and the third side will always be longer than the other equal side.)
If the equal sides were both equal to 2, the third side must be shorter than $2 + 2 = 4$. The 1 possibility from the list not equal to 2 (since we cannot have three equal sides) is 3. So here there is 1 possibility.
If the equal sides were both equal to 3, the third side must be shorter than $3 + 3 = 6$. The 2 possibilities from the list not equal to 3 (since we cannot have three equal sides) are 2 and 5. So here there are 2 possibilities.
If the equal sides were both equal to 5, the third side must be shorter than $5 + 5 = 10$. The 3 possibilities from the list not equal to 5 (since we cannot have three equal sides) are 2, 3 and 7. So here there are 3 possibilities.
If the equal sides were both equal to 7, the third side must be shorter than $7 + 7 = 14$. The 4 possibilities from the list not equal to 7 (since we cannot have three equal sides) are 2, 3, 5

and 11. So here there are 4 possibilities.
If the equal sides were both equal to 11, the third side must be shorter than $11+11=22$. The 4 possibilities from the list not equal to 11 (since we cannot have three equal sides) are 2, 3, 5 and 7. So here there are 4 possibilities.
Thus, in total there are $1+2+3+4+4=14$ possibilities.

Answer: (E)

25. The five scores are N, 42, 43, 46, and 49.
If $N<43$, the median score is 43.
If $N>46$, the median score is 46.
If $N\geq 43$ and $N\leq 46$, then N is the median.
We try each case.

If $N<43$, then the median is 43, so the mean should be 43.
Since the mean is 43, then the sum of the 5 scores must be $5\times 43=215$.
Therefore, $N+42+43+46+49=215$ or $N+180=215$ or $N=35$, which is indeed less than 43.
We can check that the median and mean of 35, 42, 43, 46 and 49 are both 43.

If $N>46$, then the median is 46, so the mean should be 46.
Since the mean is 46, then the sum of the 5 scores must be $5\times 46=230$.
Therefore, $N+42+43+46+49=230$ or $N+180=230$ or $N=50$, which is indeed greater than 46.
We can check that the median and mean of 42, 43, 46, 49, and 50 are both 46.

If $N\geq 43$ and $N\leq 46$, then the median is N, so the mean should be N.
Since the mean is N, then the sum of the 5 scores must be $5N$.
Therefore, $N+42+43+46+49=5N$ or $N+180=5N$ or $4N=180$, or $N=45$, which is indeed between 43 and 46.
We can check that the median and mean of 42, 43, 45, 46 and 49 are both 45.

Therefore, there are 3 possible values for N.

Answer: (A)

Grade 8

1. Calculating using the correct order of operations, $30 - 5^2 = 30 - 25 = 5$.

   Answer: (E)

2. *Solution 1*
   Since $98 \div 2 = 49$, $98 \div 7 = 14$, $98 \div 14 = 7$, $98 \div 49 = 2$, and $98 \div 4 = 24.5$, then the one of the five choices which does not divide exactly into 98 is 4.

   Solution 2
   The prime factorization of 98 is $98 = 2 \times 7 \times 7$.
   Of the given possibilities, only $4 = 2 \times 2$ cannot be a divisor of 98, since there are not two 2's in the prime factorization of 98.

   Answer: (B)

3. Since the tax is 15% on the \$200.00 camera, then the tax is $0.15 \times \$200.00 = \$30.00$.

   Answer: (A)

4. Since $1 + 1.1 + 1.11 = 3.21$, then to get a sum of 4.44, we still need to add $4.44 - 3.21 = 1.23$. Thus, the number which should be put in the box is 1.23.

   Answer: (B)

5. In total, there are $2 + 5 + 4 = 11$ balls in the bag.
   Since there are 5 yellow balls, then the probability of choosing a yellow ball is $\frac{5}{11}$.

   Answer: (B)

6. We check each number between 20 and 30.
   None of 20, 22, 24, 26, 28, and 30 is a prime number, because each has a factor of 2.
   Neither 21 nor 27 is a prime number, since each has a factor of 3.
   Also, 25 is not a prime number, since it has a factor of 5.
   We do not need to look for any larger prime factors, since this leaves only 23 and 29, each of which is a prime number. Therefore, there are 2 prime numbers between 20 and 30.

   Answer: (C)

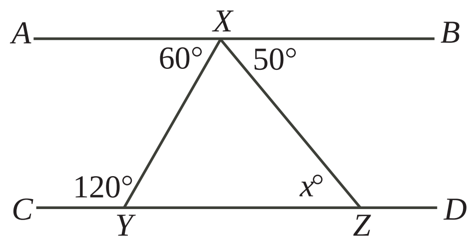
7. Since the volume of a rectangular block is equal to the area of its base times its height, then the height of this particular rectangular block is $\dfrac{120 \text{ cm}^3}{24 \text{ cm}^2} = 5$ cm.

   Answer: (A)

8. Since the rate of rotation of the fan doubles between the slow and medium settings and the fan rotates 100 times in 1 minute on the slow setting, then it rotates $2 \times 100 = 200$ times in 1 minute on the medium setting.
   Since the rate of rotation of the fan doubles between the medium and high setting, then it rotates $2 \times 200 = 400$ times in 1 minute on the high setting.
   Therefore, in 15 minutes, it will rotate $15 \times 400 = 6000$ times.

   Answer: (C)

9. *Solution 1*



Since $\angle AXB = 180^\circ$, then $\angle YXZ = 180^\circ - 60^\circ - 50^\circ = 70^\circ$.
Also, $\angle XYZ = 180^\circ - \angle CYX = 180^\circ - 120^\circ = 60^\circ$.
Since the angles in $\triangle XYZ$ add to 180°, then $x^\circ = 180^\circ - 70^\circ - 60^\circ = 50^\circ$, so $x = 50$.

Solution 2
Since $\angle CYX + \angle AXY = 180^\circ$, then AB is parallel to CD.
Therefore, $\angle YZX = \angle ZXB$ or $x^\circ = 50^\circ$ or $x = 50$.

Answer: (A)

10. *Solution 1*
When we divide 8362 by 12, we obtain

$$\begin{array}{r@{\,}l} & 696 \\ 12 & \overline{)8362} \\ & \underline{72} \\ & 116 \\ & \underline{108} \\ & 82 \\ & \underline{72} \\ & 10 \end{array}$$

so $\dfrac{8362}{12} = 696\dfrac{10}{12}$. In other words, 8362 lollipops make up 696 complete packages leaving 10 lollipops left over.

Solution 2
When we divide 8362 by 12, we obtain approximately 696.83, so the maximum possible number of packages which can be filled is 696.
In total, 696 packages contain 8352 lollipops, leaving 10 remaining from the initial 8362 lollipops.

Answer: (E)

11. Since the sound of thunder travels at 331 m/s and the thunder is heard 12 seconds after the lightning flash, then Joe is 12 s $\times$ 331 m/s = 3972 m = 3.972 km from the lightning flash, or, to the nearest tenth of a kilometre, 4.0 km.

Answer: (C)

12. The shaded triangle has a base of length 10 cm.
Since the triangle is enclosed in a rectangle of height 3 cm, then the height of the triangle is 3 cm.
(We know that the enclosing shape is a rectangle, because any figure with 4 sides, including 2 pairs of equal opposite sides, and 2 right angles must be a rectangle.)
Therefore, the area of the triangle is $\frac{1}{2} \times 3 \times 10 = 15$ cm^2.

Answer: (C)

13. We need to find two consecutive numbers the first of which is a multiple of 7 and the second of which is a multiple of 5.
We try the multiples of 7 and the numbers after each.
Do 7 and 8 work? No, since 8 is not a multiple of 5.
Do 14 and 15 work? Yes, since 15 is a multiple of 5.
This means that this year, Kiril is 15 years old.
So it will be 11 years until Kiril is 26 years old.
(Of course, Kiril could also be 50 years old or 85 years old this year, but then he would have already been 26 years old in the past.)

Answer: (A)

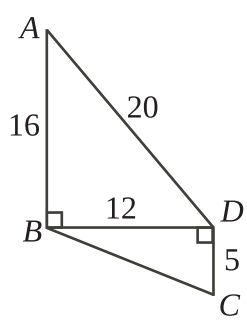
14. We are told that the second term in the sequence is 260.
Using the rule for the sequence, to get the third term, we divide the second term by 2 to obtain 130 and then add 10 to get 140, which is the third term.
To get the fourth term, we divide the third term by 2 to obtain 70 and then add 10 to get 80.

Answer: (E)

15. When the original shape  is reflected in Line 1 (that is, reflected vertically), the shape  is obtained.
When this new shape is reflected in Line 2 (that is, reflected horizontally), the shape that results is  .

Answer: (D)

16. By the Pythagorean Theorem in $\triangle ABD$, we have $BD^2 + 16^2 = 20^2$ or $BD^2 + 256 = 400$ or $BD^2 = 144$. Therefore, $BD = 12$.



By the Pythagorean Theorem in $\triangle BDC$, we have $BC^2 = 12^2 + 5^2 = 144 + 25 = 169$, so $BC^2 = 169$ or $BC = 13$.

Answer: (A)

17. Since $10^x - 10 = 9990$, then $10^x = 9990 + 10 = 10\,000$.
If $10^x = 10\,000$, then $x = 4$, since $10\,000$ ends in 4 zeroes.

Answer: (D)

18. Since the square has perimeter 24, then the side length of the square is $\frac{1}{4} \times 24 = 6$.
Since the square has side length 6, then the area of the square is $6^2 = 36$.
Since the rectangle and the square have the same area, then the area of rectangle is 36.
Since the rectangle has area 36 and width 4, then the length of the rectangle is $\dfrac{36}{4} = 9$.
Since the rectangle has width 4 and length 9, then it has perimeter $4 + 9 + 4 + 9 = 26$.

Answer: (A)

19. *Solution 1*

We can write out the possible arrangements, using the first initials of the four people, and remembering that there two different ways in which Dominic and Emily can sit side by side:

DEBC, DECB, EDBC, EDCB, BDEC, CDEB, BEDC, CEDB, BCDE, CBDE, BCED, CBED

There are 12 possible arrangements.

Solution 2

Since Dominic and Emily must sit beside other, then we can combine then into one person – either Domily or Eminic, depending on the order in which they sit – and their two seats into one seat.

We now must determine the number of ways of arranging Bethany, Chun and Domily into three seats, and the number of ways of arranging Bethany, Chun and Eminic into three seats.

With three people, there are three possible choices for who sits in the first seat, and for each of these choices there are two possible choices for the next seat, leaving the choice for the final seat fixed. So with three people, there are $3 \times 2 = 6$ possible ways that they can sit in three chairs.

So for each of the 2 ways in which Dominic and Emily can sit, there are 6 ways that the seating can be done, for a total of 12 seating arrangements.

Answer: (C)

20. We label the blank spaces to make them easier to refer to.

$$\begin{array}{cccc} & & \boxed{A} & \boxed{B} \\ + & & \boxed{C} & \boxed{D} \\ \hline & \boxed{E} & \boxed{F} & \boxed{G} \end{array}$$

Since we are adding two 2-digit numbers, then their sum cannot be 200 or greater, so E must be a 1 if the sum is to have 3 digits.

Where can the digit 0 go?

Since no number can begin with a 0, then neither A nor C can be 0.

Since each digit is different, then neither B and D can be 0, otherwise both D and G or B and G would be the same.

Therefore, only F or G could be 0.

Since we are adding two 2-digit numbers and getting a number which is at least 100, then $A+C$ must be at least 9. (It could be 9 if there was a "carry" from the sum of the units digits.) This tells us that A and C must be 3 and 6, 4 and 5, 4 and 6, or 5 and 6.

If G was 0, then B and D would have to 4 and 6 in some order. But then the largest that A and C could be would be 3 and 5, which are not among the possibilities above.

Therefore, G is not 0, so $F = 0$.

$$\begin{array}{rccc} & & \boxed{A} & \boxed{B} \\ + & & \boxed{C} & \boxed{D} \\ \hline & \boxed{1} & \boxed{0} & \boxed{G} \end{array}$$

So the sum of A and C is either 9 or 10, so A and C are 3 and 6, 4 and 5, or 4 and 6.
In any of these cases, the remaining possibilities for B and D are too small to give a carry from the units column to the tens column.
So in fact, A and C must add to 10, so A and C are 4 and 6 in some order.
Let's try $A = 4$ and $C = 6$.

$$\begin{array}{rccc} & & \boxed{4} & \boxed{B} \\ + & & \boxed{6} & \boxed{D} \\ \hline & \boxed{1} & \boxed{0} & \boxed{G} \end{array}$$

The remaining digits are 2, 3 and 5. To make the addition work, B and D must be 2 and 3 and G must be 5. (We can check that either order for B and D works, and that switching the 4 and 6 will also work.)

So the units digit of the sum must be 5, as in the example

$$\begin{array}{rccc} & & \boxed{4} & \boxed{2} \\ + & & \boxed{6} & \boxed{3} \\ \hline & \boxed{1} & \boxed{0} & \boxed{5} \end{array}$$

(Note that we could have come up with this answer by trial and error instead of this logical procedure.)

ANSWER: (D)

21. *Solution 1*
If Nathalie had exactly 9 quarters, 3 dimes and 1 nickel, she would have
$9 \times 25 + 3 \times 10 + 5 = 260$ cents or \$2.60 in total.
Since the coins she has are in this same ratio $9 : 3 : 1$, then we can split her coins up into sets of 9 quarters, 3 dimes and 1 nickel, with each set having a value of \$2.60.
Since the total value of her coins is \$18.20 and $\dfrac{\$18.20}{\$2.60} = 7$, then she has 7 of these sets of coins.
Since each of these sets contains 13 coins, then she has $7 \times 13 = 91$ coins in total.

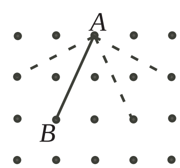
Solution 2
Suppose Nathalie had n nickels. Since the ratio of the number of quarters to the number of dimes to the number of nickels that she has is $9 : 3 : 1$, then she must have $3n$ dimes and $9n$ quarters.
This tells us that the total value of her coins in cents is $(9n \times 25) + (3n \times 10) + (n \times 5) = 260n$.
Since we know that the total value of her coins is 1820 cents, then $260n = 1820$ or $n = 7$.
Therefore, she has 7 nickels, 21 dimes and 63 quarters, or 91 coins in total.

ANSWER: (D)

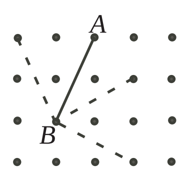
22. Label the first 8 people at the party as A, B, C, D, E, F, G, and H.
Then A shakes hands with the 7 others, B accounts for 6 more handshakes (since we have already included B shaking hands with A), C accounts for 5 more handshakes (since we have already included C shaking hands with A and B), D accounts for 4 more handshakes, E accounts for 3 more handshakes, F accounts for 2 more handshakes, and G accounts for 1 more handshake. (H's handshakes with each of the others have already been included.)
So there are $7+6+5+4+3+2+1 = 28$ handshakes which take place before the ninth person arrives.
Since there are a total of 32 handshakes which take place, then the ninth person shakes hands with $32 - 28 = 4$ people.

Answer: (B)

23. In how many ways can we place C so that $AB = AC$?
To get from A to B we go 1 unit in one direction and 2 units in the perpendicular direction from A.
To choose C so that $AB = AC$, we must also go 1 unit in one direction and 2 units in the perpendicular direction.
This gives the 3 possible points:



In how many ways can we place C so that $BA = BC$?
To choose C so that $BA = BC$, we again also go 1 unit in one direction and 2 units in the other direction from B.
This gives the 3 possible points:



So in total we have 6 possible points so far, which is the largest of the possible answers.
So 6 must be the answer.
(There is one more possibility: can C be placed so that $CA = CB$? Since 6 must be the answer, then the answer to this question is no. We can also check this by trying to find such points in the diagram.)

Answer: (A)

24. In the 1st row, every box is shaded.
In the 2nd row, the boxes whose column numbers are multiples of 2 are shaded.
In the 3rd row, the boxes whose column numbers are multiples of 3 are shaded.
In the nth row, the boxes whose column numbers are multiples of n are shaded.
So in a particular column, the boxes which are shaded are those which belong to row numbers which are divisors of the column number.
(We can see this for instance in columns 4 and 6 where the boxes in rows 1, 2 and 4, and 1, 2,

3, and 6, respectively, are shaded.)
So to determine which of the given columns has the largest number of shaded boxes, we must determine which of the given numbers has the greatest number of divisors.
To find the divisors of 144, for instance, it is easier to find the prime factors of 144 first.
To do this, we see that $144 = 16 \times 9 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$.
We can then write out the divisors of 144, which are 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144, or a total of 15 divisors.
Similarly, $120 = 2^3 \times 3 \times 5$, so we can find the divisors of 120, which are 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120, or a total of 16 divisors.
Since $150 = 2 \times 3 \times 5^2$, the divisors of 150 are 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150, or a total of 12 divisors.
Since $96 = 2^5 \times 3$, the divisors of 96 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96, or a total of 12 divisors.
Since $100 = 2^2 \times 5^2$, the divisors of 100 are 1, 2, 4, 5, 10, 20, 25, 50, 100, or a total of 9 divisors.
So the number with the most divisors (and thus the column with the most shaded boxes) is 120.

Answer: (B)

25. The numbers which have yet to be placed in the grid are 10, 11, 12, 13, 14, 15, 16, 17, 18, and 19.
We look at the numbers in the corners first, because they have the fewest neighbours.
Look first at the 25 in the bottom left corner. It must be the sum of two of its neighbours, so it must be $24 + 1$ or $9 + 16$. Since the 1 has already been placed in the grid, then the empty space next to the 25 must be filled with the 16.

Look next at the 21 in the top right corner. It must be equal to $20 + 1$ (which it cannot be since the 1 has already been placed) or $4 + 17$, so the 17 must be placed in the space next to the 21.

			20	21
	6	5	4	17
23	7	1	3	?
16	9	8	2	
25	24			22

Since the 17 must be the sum of two of its neighbours, then the 17 must be $4 + 13$ or $3 + 14$, so the "?" must be replaced by either the 13 or the 14.

Consider the 22 in the corner. Since the 20 is already placed, 22 cannot be $2 + 20$. So the 22 is the sum of its two missing neighbours.
Since the possible missing neighbours are 10, 11, 12, 13, 14, 15, and 18, then the two neighbours must be 10 and 12 in some order.
However, the 10 cannot go above the 22, since there it could not be the sum of two of its neighbours (since the 7 and 8 have already been placed).
Therefore, the 12 goes above the 22, so we have

			20	21
	6	5	4	17
23	7	1	3	?
16	9	8	2	12
25	24		10	22

The 13 now cannot replace the "?" since 13 is not the sum of any two of 17, 4, 3, 2, and 12. Therefore, the "?" is replaced by the 14.

At this stage, we are already finished, but we could check that the grid does complete as follows:

19	11	15	20	21
13	6	5	4	17
23	7	1	3	14
16	9	8	2	12
25	24	18	10	22

Answer: (C)