2006 Galois Contest (Grade 10)
Thursday, April 20, 2006

1. A hat contains six slips of paper numbered from 1 to 6. Amelie and Bob each choose three slips from the hat without replacing any of the slips. Each of them adds up the numbers on his slips.

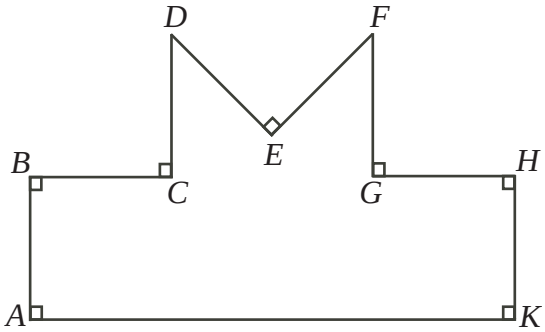
   (a) Determine the largest possible difference between Amelie's total and Bob's total. Explain how you found this difference.

   (b) List all possible groups of three slips that Amelie can choose so that her total is one more than Bob's total.

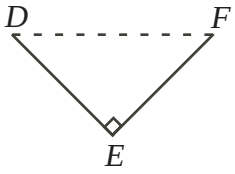
   (c) Explain why it is impossible for Amelie and Bob to have the same total no matter which three slips each chooses.

   (d) If more slips of paper are added to the hat, numbered consecutively from 7 to n, what is the smallest value of $n > 6$ so that Amelie and Bob can each choose half of the slips numbered from 1 to n and obtain the same total? Explain why this value of n works.

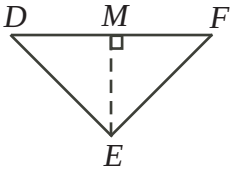
2. In the diagram, AB, BC, CD, DE, EF, FG, GH, and HK all have length 4, and all angles are right angles, with the exception of the angles at D and F.



   (a) Determine the length of DF.



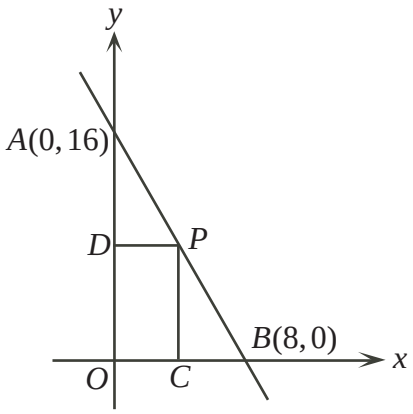
   (b) If perpendicular EM is drawn from E to DF, what is the length of EM? Explain how you got your answer.



   (c) If perpendicular EP is drawn from E to AK, what is the length of EP? Explain how you got your answer.

   (d) What is the area of figure $ABCDEFGHK$? Explain how you got your answer.

3. In the diagram, a line is drawn through the points $A(0, 16)$ and $B(8, 0)$. Point P is chosen in the first quadrant on the line through A and B. Points C and D are then chosen on the x-axis and y-axis, respectively, so that $PDOC$ is a rectangle.



   (a) Determine the equation of the line through A and B.

   (b) Determine the coordinates of the point P so that $PDOC$ is a square.

   (c) Determine the coordinates of all points P that can be chosen so that the area of rectangle $PDOC$ is 30.

4. (a) When the number 14 has its digits reversed to form the number 41, it is increased by 27. Determine all 2-digit numbers which are increased by 27 when their digits are reversed.

   (b) Choose any three-digit integer $\underline{a}\,\underline{b}\,\underline{c}$ whose digits are all different.
   (When a three-digit integer is written in terms of its digits as $\underline{a}\,\underline{b}\,\underline{c}$, it means the integer is $100a + 10b + c$.)
   Reverse the order of the digits to get a new three-digit integer $\underline{c}\,\underline{b}\,\underline{a}$.
   Subtract the smaller of these integers from the larger to obtain a three-digit integer $\underline{r}\,\underline{s}\,\underline{t}$, where r is allowed to be 0.
   Reverse the order of the digits of this integer to get the integer $\underline{t}\,\underline{s}\,\underline{r}$.
   Prove that, no matter what three-digit integer $\underline{a}\,\underline{b}\,\underline{c}$ you start with, $\underline{r}\,\underline{s}\,\underline{t} + \underline{t}\,\underline{s}\,\underline{r} = 1089$.

   (c) Suppose that $N = \underline{a}\,\underline{b}\,\underline{c}\,\underline{d}$ is a four-digit integer with $a \leq b \leq c \leq d$.
   When the order of the digits of N is reversed to form the integer M, N is increased by P. (Again, the first digit of P is allowed to be 0.)
   When the order of the digits of P is reversed, an integer Q is formed.
   Determine, with justification, all possible values of $P + Q$.