Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest *(Grade 10)*

Wednesday, February 22, 2006

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{2}\right)$ is

   (A) $\frac{3}{8}$ **(B)** 1 **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{3}{4}$

2. The value of $\left(\sqrt{100} - \sqrt{36}\right)^2$ is

   (A) 16 **(B)** 256 **(C)** 8 **(D)** 1024 **(E)** 4096

3. The value of $43 - 41 + 39 - 37 + 35 - 33 + 31 - 29$ is

   (A) 8 **(B)** 6 **(C)** 10 **(D)** 12 **(E)** 16

4. If $a = -3$ and $b = 2$, the value of $a(b - 3)$ is

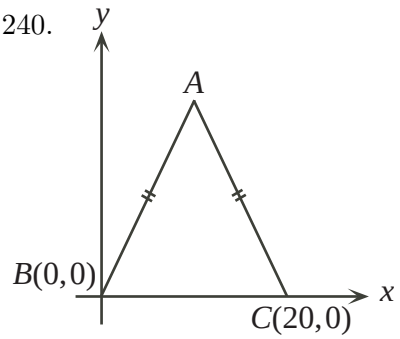
   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

5. In the four term sequence $0.001,\ 0.02,\ 0.4,\ x$, each term after the first is found by multiplying the previous term by the same number. What is the value of x?

   (A) 0.8 **(B)** 8 **(C)** 80 **(D)** 8.8 **(E)** 0.08

6. In the diagram, $\triangle ABC$ is isosceles and its area is 240. The y-coordinate of A is

   (A) 6 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 48

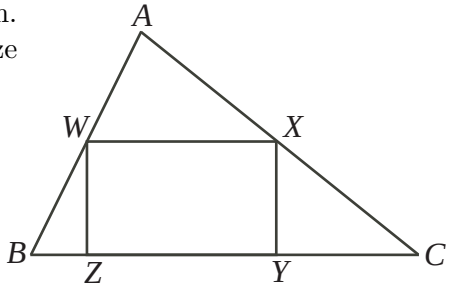


7. If $\dfrac{6}{x+1} = \dfrac{3}{2}$, then x equals

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8. A rectangle is drawn inside $\triangle ABC$, as shown. If $\angle BWZ = 22°$ and $\angle CXY = 65°$, then the size of $\angle BAC$ is

   (A) 87° **(B)** 90° **(C)** 93° **(D)** 104° **(E)** 82°



9. The lengths of the three sides of a triangle are 7, $x + 4$ and $2x + 1$. The perimeter of the triangle is 36. What is the length of the longest side of the triangle?

   (A) 7 **(B)** 12 **(C)** 17 **(D)** 15 **(E)** 16

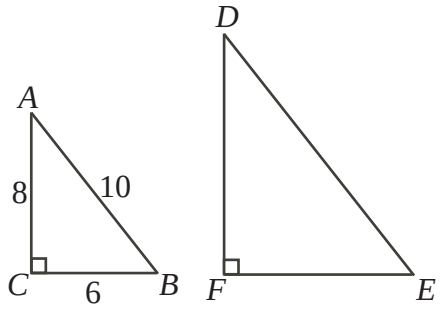
10. A class of 30 students recently wrote a test. If 20 students scored 80, 8 students scored 90, and 2 students scored 100, then the class average on this test was

(A) 90 **(B)** 84 **(C)** 82 **(D)** 86 **(E)** 88

Part B: Each correct answer is worth 6.

11. $\triangle ABC$ has side lengths 6, 8 and 10, as shown. Each of the side lengths of $\triangle ABC$ is increased by 50%, forming a new triangle, $\triangle DEF$. The area of $\triangle DEF$ is

(A) 24 **(B)** 48 **(C)** 108 **(D)** 12 **(E)** 54



12. From 7:45 p.m. to 9:30 p.m., Jim drove a distance of 84 km at a constant speed. What was this speed, in km/h?

(A) 60 **(B)** 80 **(C)** 112 **(D)** 63 **(E)** 48

13. If $x + 1 = y - 8$ and $x = 2y$, then the value of $x + y$ is

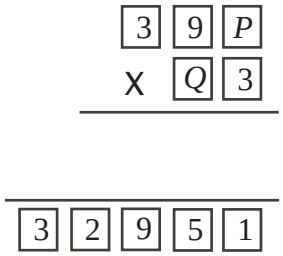
(A) -18 **(B)** 0 **(C)** -9 **(D)** -27 **(E)** -36

14. If $x = -3$, which of the following expressions has the smallest value?

(A) $x^2 - 3$ **(B)** $(x - 3)^2$ **(C)** x^2 **(D)** $(x + 3)^2$ **(E)** $x^2 + 3$

15. In the multiplication shown, P and Q each represent a single digit, and the product is 32 951. What is the value of $P + Q$?

(A) 14 **(B)** 12 **(C)** 15 **(D)** 13 **(E)** 11



16. In 2004, Gerry downloaded 200 songs. In 2005, Gerry downloaded 360 songs at a cost per song which was 32 cents less than in 2004. Gerry's *total* cost each year was the same. The cost of downloading the 360 songs in 2005 was

(A) \$144.00 **(B)** \$108.00 **(C)** \$80.00 **(D)** \$259.20 **(E)** \$72.00

17. If w is a positive integer and $w^3 = 9w$, then w^5 is equal to

(A) 59 049 **(B)** 243 **(C)** 1024 **(D)** 3125 **(E)** 32

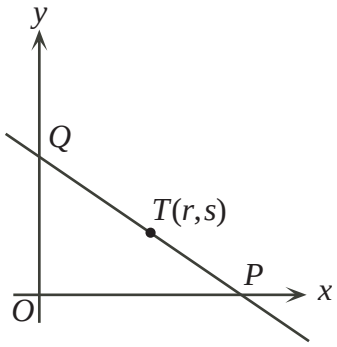
18. In a right-angled triangle, the sum of the squares of the three side lengths is 1800. The length of its hypotenuse is

(A) $\sqrt{1800}$ **(B)** $\frac{1}{2}\sqrt{1800}$ **(C)** 90 **(D)** 30 **(E)** 45

19. In a bin at the Cayley Convenience Store, there are 200 candies. Of these candies, 90% are black and the rest are gold. After Yehudi eats some of the black candies, 80% of the remaining candies in the bin are black. How many black candies did Yehudi eat?

(A) 2 **(B)** 20 **(C)** 40 **(D)** 100 **(E)** 160

20. The line $y = -\frac{3}{4}x + 9$ crosses the x-axis at P and the y-axis at Q. Point $T(r, s)$ is on line segment PQ. If the area of $\triangle POQ$ is three times the area of $\triangle TOP$, then the value of $r + s$ is



(A) 7 **(B)** 10 **(C)** 11 **(D)** 14 **(E)** 18

Part C: Each correct answer is worth 8.

21. If p, q and r are positive integers and $p + \dfrac{1}{q + \dfrac{1}{r}} = \dfrac{25}{19}$, then q equals

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

22. A positive integer is called *multiplicatively perfect* if it is equal to the product of its proper divisors. For example, 10 is multiplicatively perfect since its proper divisors are 1, 2 and 5, and it is true that $1 \times 2 \times 5 = 10$. How many multiplicatively perfect integers are there between 2 and 30?

(A) 9 **(B)** 5 **(C)** 8 **(D)** 6 **(E)** 4

23. Quincy and Celine have to move 16 small boxes and 10 large boxes. The chart indicates the time that each person takes to move each type of box. They start moving the boxes at 9:00 a.m. The earliest time at which they can be finished moving all of the boxes is

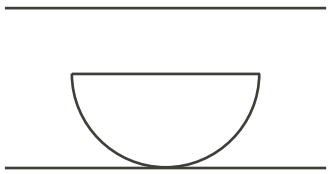
	Celine	Quincy
small box	2 min.	3 min.
large box	6 min.	5 min.

(A) 9:41 a.m. **(B)** 9:42 a.m. **(C)** 9:43 a.m. **(D)** 9:44 a.m. **(E)** 9:45 a.m.

24. Anne and Brenda play a game which begins with a pile of n toothpicks. They alternate turns with Anne going first. On each player's turn, she must remove 1, 3 or 4 toothpicks from the pile. The player who removes the last toothpick wins the game. For which of the following values of n does Brenda have a winning strategy? (In a game, a player has a winning strategy if, regardless of what the other player does, there are moves that she can make which *guarantee* that she will win.)

(A) 31 **(B)** 32 **(C)** 33 **(D)** 34 **(E)** 35

25. A semi-circle of radius 8 cm, rocks back and forth along a line. The distance between the line on which the semi-circle sits and the line above is 12 cm. As it rocks without slipping, the semi-circle touches the line above at two points. (When the semi-circle hits the line above, it immediately rocks back in the other direction.) The distance between these two points, in millimetres, is closest to



(A) 55 **(B)** 53 **(C)** 51

(D) 49 **(E)** 47

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