



# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## *Pascal Contest* (Grade 9)

Wednesday, February 18, 2004

C.M.C. Sponsors:



**Deloitte  
& Touche**  
Chartered Accountants

C.M.C. Supporters:



Canadian Institute  
of Actuaries

**Great West Life**  
ASSURANCE COMPANY



Great West Life  
and London Life

**SYBASE**

Sybase  
Inc. (Waterloo)

**iAnywhere**  
A SYBASE COMPANY

iAnywhere Solutions

**Time:** 1 hour

© 2003 Waterloo Mathematics Foundation

**Calculators are permitted.**

### Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

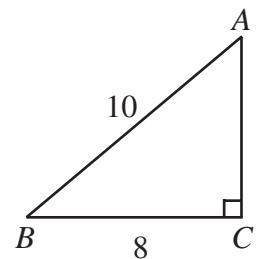
**Part A: Each correct answer is worth 5.**

1. To win a skateboard, the skill testing question is  $5 \times (10 - 6) \div 2$ . The correct answer is  
(A) 10            (B) 35            (C) 32            (D) 22            (E) 40

2. The average of 2,  $x$  and 12 is 8. What is the value of  $x$ ?  
(A) 8            (B) -2            (C) 12            (D) 24            (E) 10

3. The fractions  $\frac{1}{9}$ ,  $\frac{1}{4}$  and  $\frac{1}{18}$  are to be added. What is their lowest common denominator?  
(A) 648            (B) 162            (C) 72            (D) 36            (E) 18

4. In the diagram, the area of  $\triangle ABC$  is  
(A) 40            (B) 12            (C) 30  
(D) 48            (E) 24



5. The value of  $\frac{5 - \sqrt{4}}{5 + \sqrt{4}}$  is  
(A)  $\frac{3}{7}$             (B)  $\frac{1}{9}$             (C)  $-\frac{11}{21}$             (D) 0            (E)  $\frac{1}{3}$

6. The value of  $4^1 + 3^2 - 2^3 + 1^4$  is  
(A) 4            (B) 8            (C) 6            (D) 5            (E) 9

7. When  $x = -3$ , the value of  $3x^2 + 2x$  is  
(A) 81            (B) 75            (C) -33            (D) 21            (E) -24

8. If 18% of 42 is equal to 27% of  $x$ , then the value of  $x$  is  
(A) 28            (B) 63            (C) 2            (D) 864            (E) 12

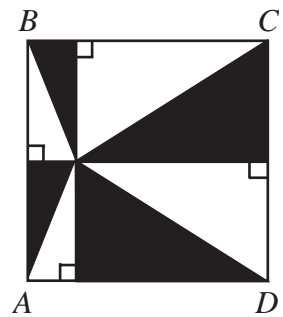
9. The surface area of a cube is  $96 \text{ cm}^2$ . The volume of the cube, in  $\text{cm}^3$ , is  
(A) 16            (B) 64            (C) 8            (D) 512            (E) 216

10. It is given that  $y = 3x - 5$  and  $z = 3x + 3$ . If  $y = 1$ , the value of  $z$  is  
(A) 8            (B) 6            (C) -3            (D) 3            (E) 9

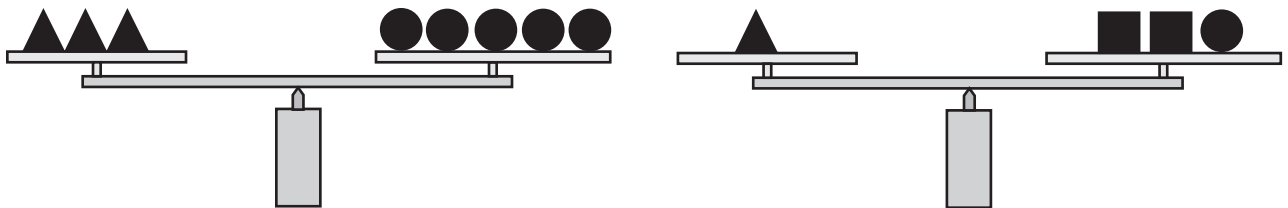
**Part B: Each correct answer is worth 6.**

11. In the diagram, square  $ABCD$  has a side length of 4. What is the total area of the shaded regions?

- (A) 4                      (B) 8                      (C) 9  
 (D) 12                     (E) 16

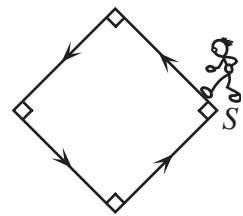


12. In the diagram, two equal-armed balances are shown. How many would it take to balance one ?



- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

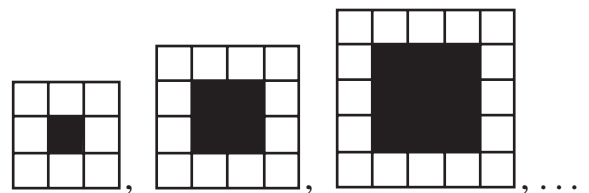
13. Nadia starts at  $S$  and walks at a steady pace once around the perimeter of a square park. Which graph best represents her distance from  $S$  as time passes?



- (A) distance vs time: A graph showing a linear increase, a constant distance, and a linear decrease.  
 (B) distance vs time: A graph showing a linear increase, a linear decrease, a linear increase, and a linear decrease.  
 (C) distance vs time: A graph showing a smooth, concave-down curve that starts at the origin and ends at the same height.  
 (D) distance vs time: A graph showing a linear increase, a small wave-like fluctuation, and a linear decrease.  
 (E) distance vs time: A graph showing a linear increase, a smooth curve that dips and rises, and a linear decrease.

14. How many unshaded squares are in the tenth figure of the pattern?

- (A) 38                      (B) 40                      (C) 42  
 (D) 44                      (E) 46



15. In the Pascal family, each child has at least 2 brothers and at least 1 sister. What is the smallest possible number of children in this family?

- (A) 3                      (B) 4                      (C) 5                      (D) 6                      (E) 7

16. If  $a^2 + 3b = 33$ , where  $a$  and  $b$  are positive integers, what is the value of  $ab$ ?

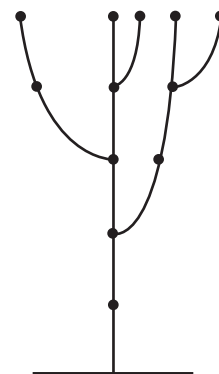
- (A) 11                      (B) 24                      (C) 16                      (D) 32                      (E) 27

17. The value of  $0.\overline{1} + 0.\overline{12} + 0.\overline{123}$  is  
 (A)  $0.\overline{343}$       (B)  $0.\overline{355}$       (C)  $0.3\overline{5}$       (D)  $0.\overline{355446}$       (E)  $0.\overline{355445}$

18. The symbol  $\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array}$  equals  $ad - bc$ . If  $\begin{array}{|c|c|} \hline x-1 & 2 \\ \hline 3 & -5 \\ \hline \end{array} = 9$ , the value of  $x$  is  
 (A)  $-4$       (B)  $-3$       (C)  $-2$       (D)  $2$       (E)  $4$

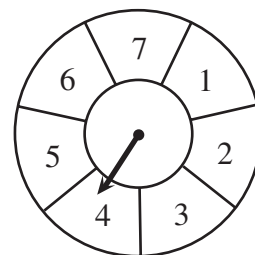
19. Raffaello's tree grows according to the following rule. After a branch has been growing for two weeks, it produces a new branch every week, while the original branch continues to grow. The tree has five branches after five weeks, as shown. How many branches, including the main branch, will the tree have at the end of eight weeks?

- (A) 21      (B) 40      (C) 19  
 (D) 13      (E) 34



20. At the beginning of the game "Clock 7", the arrow points to one of the seven numbers. On each turn, the arrow is rotated clockwise by the number of spaces indicated by the arrow at the beginning of the turn. For example, if "Clock 7" starts with the arrow pointing at 4, then on the first turn, the arrow is rotated clockwise 4 spaces so that it now points at 1. The arrow will then move 1 space on the next turn, and so on. If the arrow points at 6 after the 21st turn, at which number did the arrow point after the first turn?

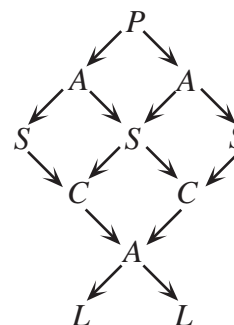
- (A) 3      (B) 6      (C) 5  
 (D) 2      (E) 7



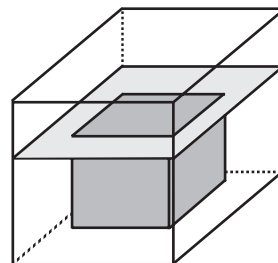
**Part C: Each correct answer is worth 8.**

21. In the diagram, the number of different paths that spell "PASCAL" is

- (A) 6      (B) 10      (C) 12  
 (D) 16      (E) 24



22. A container in the shape of a cube has edge length 20 cm and contains some water. A solid gold cube, with edge length 15 cm, sinks to the bottom of this container, causing the water level to rise just to the top of the *solid* cube. Which of the following is closest to the original depth of the water?



- (A) 6.56 cm      (B) 8.25 cm      (C) 10.50 cm  
 (D) 5.31 cm      (E) 7.50 cm

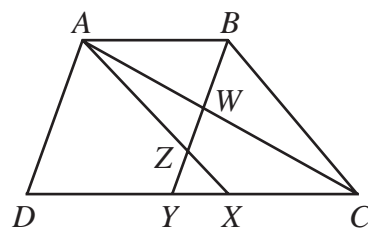
23. A driver approaching a toll booth has exactly two quarters, two dimes and two nickels in his pocket. He reaches into his pocket and randomly selects two of these coins. What is the probability that the coins that he selects will be at least enough to pay the 30-cent toll?

- (A)  $\frac{3}{5}$       (B)  $\frac{2}{5}$       (C)  $\frac{1}{3}$       (D)  $\frac{3}{10}$       (E)  $\frac{2}{3}$

24. In the sequence of fractions  $\frac{1}{1}, \frac{2}{1}, \frac{1}{2}, \frac{3}{1}, \frac{2}{2}, \frac{1}{3}, \frac{4}{1}, \frac{3}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{1}, \dots$ , fractions equivalent to any given fraction occur many times. For example, fractions equivalent to  $\frac{1}{2}$  occur for the first two times in positions 3 and 14. In which position is the fifth occurrence of a fraction equivalent to  $\frac{3}{7}$ ?

- (A) 1207      (B) 1208      (C) 1209      (D) 1210      (E) 1211

25. In the diagram,  $ABCD$  is a trapezoid with  $AB$  parallel to  $CD$  and with  $AB = 2$  and  $CD = 5$ . Also,  $AX$  is parallel to  $BC$  and  $BY$  is parallel to  $AD$ . If  $AX$  and  $BY$  intersect at  $Z$ , and  $AC$  and  $BY$  intersect at  $W$ , the ratio of the area of  $\Delta AZW$  to the area of trapezoid  $ABCD$  is



- (A) 7 : 105      (B) 8 : 105      (C) 9 : 105  
 (D) 10 : 105      (E) 12 : 105

## PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

### **Copies of Previous Canadian Mathematics Competitions**

Copies of previous contests and solutions are available at no cost in both English and French at <http://www.cemc.uwaterloo.ca>

### **Problems Problems Problems Books**

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is \$15. (**Available in English only.**)

#### **Volume 1**

- over 300 problems and full solutions
- 10 topics
- for students in Grades 9, 10, & 11
- French version of Volume 1 is available

#### **Volume 3**

- over 235 problems and full solutions
- 12 topics
- for senior high school students

#### **Volume 5**

- over 200 problems and full solutions
- 9 topics (different from Volume 3)
- for senior high school students

#### **Volume 7**

- over 300 problems and full solutions
- 12 topics
- for students in Grades 9 and 10

#### **Volume 9**

- over 300 problems and full solutions
- 11 topics
- for students in Grades 7 and 8

#### **Volume 2**

- over 325 problems and full solutions
- 10 topics (different from Volume 1)
- for students in Grades 9, 10, & 11

#### **Volume 4**

- over 325 problems and full solutions
- 12 topics
- for students in Grades 7, 8, & 9

#### **Volume 6**

- over 300 problems and full solutions
- 11 topics
- for students in Grades 7, 8, & 9

#### **Volume 8**

- over 200 problems and full solutions
- 10 topics
- for students in Grades 11 and 12

Orders should be addressed to: Canadian Mathematics Competition  
Faculty of Mathematics, Room 5181  
University of Waterloo  
Waterloo, ON N2L 3G1

Include your name, address (with postal code), and telephone number.

Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add \$3.00 for the first item ordered for shipping and handling, plus \$1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders *outside of Canada ONLY*, add \$10.00 for the first item ordered for shipping and handling, plus \$2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2004.**

**NOTE: All publications are protected by copyright. It is unlawful to make copies without the prior written permission of the Waterloo Mathematics Foundation.**

