



2004 Gauss Contest - Grade 8

Solutions

Part A

1. 25% of 2004 is $\frac{1}{4}$ of 2004, or 501.

Answer: (B)

2. Using a common denominator,

$$\frac{1}{2} + \frac{3}{4} - \frac{5}{8} = \frac{4}{8} + \frac{6}{8} - \frac{5}{8} = \frac{5}{8}$$

Answer: (C)

3. Rewriting the given integer,

$$800\,670 = 800\,000 + 600 + 70 = 8 \times 10^5 + 6 \times 10^2 + 7 \times 10^1$$

so $x = 5$, $y = 2$ and $z = 1$, which gives $x + y + z = 8$.

Answer: (B)

4. Rewriting the right side with a common denominator,

$$\frac{7863}{13} = \frac{604 \times 13 + \square}{13} = \frac{7852 + \square}{13}$$

Therefore, $\square = 11$.

Answer: (A)

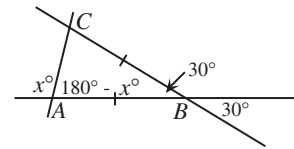
5. In the diagram, $\angle ABC = \angle XBY = 30^\circ$ since they are opposite angles.

Also, $\angle BAC = 180^\circ - x^\circ$ by supplementary angles, and so $\angle BCA = 180^\circ - x^\circ$ because triangle ABC is isosceles. Looking at the sum of the angles in triangle ABC , we have $180^\circ - x^\circ + 180^\circ - x^\circ + 30^\circ = 180^\circ$

$$210^\circ = 2x^\circ$$

$$x = 105$$

Answer: (D)



6. Since the perimeter of each of the small equilateral triangles is 6 cm, then the side length of each of these triangles is 2 cm. Since there are three of the small triangles along each side of triangle of ABC , then the side length of triangle ABC is 6 cm, and so its perimeter is 18 cm.

Answer: (A)

7. If $x = -4$ and $y = 4$, then

$$\frac{x}{y} = \frac{-4}{4} = -1$$

$$y - 1 = 4 - 1 = 3$$

$$x - 1 = -4 - 1 = -5$$

$$-xy = -(-4)(4) = 16$$

$$x + y = -4 + 4 = 0$$

Thus, $-xy$ is the largest.

Answer: (D)

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8. When two coins are tossed, there are four equally likely outcomes: HEADS and HEADS, HEADS and TAILS, TAILS and HEADS, and TAILS and TAILS. One of these four outcomes has both coins landing as HEADS. Thus, the probability is $\frac{1}{4}$.

Answer: (E)

9. The water surface has an elevation of +180 m, and the lowest point of the lake floor has an elevation of -220 m. Therefore, the actual depth of the lake at this point is $180 - (-220) = 400$ m.

Answer: (D)

10. We make a chart of the pairs of positive integers which sum to 11 and their corresponding products:

First integer	Second integer	Product
1	10	10
2	9	18
3	8	24
4	7	28
5	6	30

so the greatest possible product is 30.

Answer: (E)

Part B

11. To walk 1.5 km, Ruth takes $\frac{1.5 \text{ km}}{5 \text{ km/h}} = 0.3 \text{ h} = 18 \text{ min}$.

Answer: (C)

12. Computing each of the first and fourth of the numbers, we have the four numbers $\sqrt{36} = 6, 35.2, 35.19$ and $5^2 = 25$. Arranging these in increasing order gives 6, 25, 35.19, 35.2, or $\sqrt{36} = 6, 5^2 = 25, 35.19, 35.2$.

Answer: (D)

13. We number the trees from 1 to 13, with tree number 1 being closest to Trina's house and tree number 13 being closest to her school. On the way to school, she puts a chalk mark on trees 1, 3, 5, 7, 9, 11, 13. On her way home, she puts a chalk mark on trees 13, 10, 7, 4, 1. This leaves trees 2, 6, 8, 12 without chalk marks.

Answer: (B)

14. The rectangular prism in the diagram is made up of 12 cubes. We are able to see 10 of these 12 cubes. One of the two missing cubes is white and the other is black. Since the four blocks of each colour are attached together to form a piece, then the middle block in the back row in the bottom layer must be white, so the missing black block is the leftmost block of the back row in the bottom layer. Thus, the leftmost block in the back row in the top layer is attached to all three of the other black blocks, so the shape of the black piece is (A). (This is the only one of the 5 possibilities where one block is attached to three other blocks.)

Answer: (A)



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15. This solid can be pictured as a rectangular prism with dimensions 4 by 5 by 6 with a rectangular prism with dimensions 1 by 2 by 4 removed. Therefore, the volume is $4 \times 5 \times 6 - 1 \times 2 \times 4 = 120 - 8 = 112$.

Answer: (B)

16. Since the number is divisible by $8 = 2^3$, by $12 = 2^2 \times 3$, and by $18 = 2 \times 3^2$, then the number must have at least three factors of 2 and two factors of 3, so the number must be divisible by $2^3 \times 3^2 = 72$. Since the number is a two-digit number which is divisible by 72, it must be 72, so it is between 60 and 79.

Answer: (D)

17. Since $2^3 = 8$ and $2^a = 8$, then $a = 3$. Since $a = 3$ and $a = 3c$, then $c = 1$.

Answer: (C)

18. Since the range is unchanged after a score is removed, then the score that we remove cannot be the smallest or the largest (since each of these occurs only once). Thus, neither the 6 nor the 10 is removed. Since the mode is unchanged after a score is removed, then the score that we remove cannot be the most frequently occurring. Thus, the score removed is not an 8.

Therefore, either a 7 or a 9 is removed.

Since we wish to increase the average, we remove the smaller of the two numbers, ie. the 7. (We could calculate that before any number is removed, the average is 7.875, if the 7 is removed, the average is 8, and if the 9 is removed, the average is 7.714.)

Answer: (B)

19. Since the numerical values of CAT and CAR are 8 and 12, then the value of R must be 4 more than the value of T.

Therefore, the value of BAR is 4 more than the value of BAT, so BAR has a numerical value of 10.

Answer: (A)

20. To get from A to E , we go right 5 and up 9, so $AE = \sqrt{5^2 + 9^2} = \sqrt{106} \approx 10.30$, by the Pythagorean Theorem.

To get from C to F , we go right 2 and down 4, so $CF = \sqrt{2^2 + 4^2} = \sqrt{20} \approx 4.47$, and so $CD + CF \approx 5 + 4.47 = 9.47$.

To get from A to C , we go right 3 and up 4, so

$$AC = \sqrt{3^2 + 4^2} = \sqrt{25} = 5, \text{ and so}$$

$$AC + CF \approx 5 + 4.47 = 9.47.$$

To get from F to D , we go left 2 and up 9, so

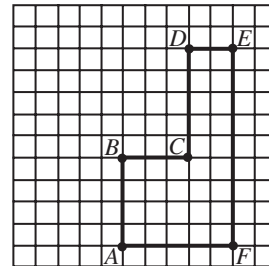
$$FD = \sqrt{2^2 + 9^2} = \sqrt{85} \approx 9.22.$$

To get from C to E , we go right 2 and up 5, so

$$CE = \sqrt{2^2 + 5^2} = \sqrt{29} \approx 5.39, \text{ and so}$$

$$AC + CE \approx 5 + 5.39 = 10.39.$$

Therefore, the longest of these five lengths is $AC + CE$.



Answer: (E)



Solutions

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Part C

21. The scale of the map is equal to the ratio of a distance on the map to the actual distance. Since the distance between Saint John and St. John's is 21 cm on the map and 1050 km in reality, then the scale of the map is equal to

$$21 \text{ cm} : 1050 \text{ km} = 0.21 \text{ m} : 1\,050\,000 \text{ m} = 21 : 105\,000\,000 = 1 : 5\,000\,000$$

Answer: (E)

22. *Solution 1*

When the pouring stops, $\frac{1}{4}$ of the water in the bottle has been transferred to the glass. This represents $\frac{3}{4}$ of the volume of the glass. Therefore, the volume of the bottle is three times the volume of the glass, so the volume of the glass is 0.5 L.

Solution 2

When the pouring stops, $\frac{1}{4}$ of the water in the bottle or $\frac{1}{4} \times 1.5 = 0.375$ L of water is in the glass. Since this represents $\frac{3}{4}$ of the volume of the glass, then the volume of the glass is $\frac{4}{3} \times 0.375 = 0.5$ L.

Answer: (A)

23. From the diagram, $BE = AD$ and $AE = CD$, so
 $AC = AD + CD = BE + AE = AB$ so triangle ABC is isosceles.

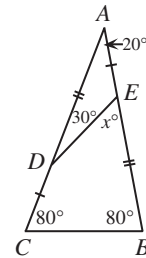
Therefore, $\angle ACB = \angle ABC = 80^\circ$ and so

$$\angle BAC = 180^\circ - \angle ABC - \angle ACB = 180^\circ - 80^\circ - 80^\circ = 20^\circ.$$

Considering triangle AED next,

$$\angle AED = 180^\circ - \angle ADE - \angle EAD = 180^\circ - 30^\circ - 20^\circ = 130^\circ.$$

$$\text{But } x^\circ = 180^\circ - \angle AED = 180^\circ - 130^\circ = 50^\circ \text{ so } x = 50.$$



Answer: (C)

24. Since x has digits ABC , then $x = 100A + 10B + C$.
 Since y has digits CBA , then $y = 100C + 10B + A$.
 Since $x - y = 495$, then

$$(100A + 10B + C) - (100C + 10B + A) = 495$$

$$99A - 99C = 495$$

$$99(A - C) = 495$$

$$A - C = 5$$

and there is no restriction on B .

Thus, there are 10 possibilities for B (0 through 9) and for each of these possibilities we could have A and C equal to 6 and 1, 7 and 2, 8 and 3, or 9 and 4. (For example, $873 - 378 = 495$.)

Therefore, there are 40 possibilities for x .

Answer: (B)



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25. Consider the block as n layers each having 11 rows and 10 columns.

First, we consider positions of the 2 by 1 by 1 block which are entirely contained in one layer. In each layer, there are 9 possible positions for the 2 by 1 by 1 block in each row (crossing columns 1 and 2, 2 and 3, 3 and 4, and so on, up to 9 and 10), and there are 10 possible positions in each column (crossing rows 1 and 2, 2 and 3, 3 and 4, and so on, up to 10 and 11). Therefore, within each layer, there are $11(9) + 10(10) = 199$ positions for the 2 by 1 by 1 block. In the large block, there are thus $199n$ positions of this type for the 2 by 1 by 1 block, since there are n layers.

Next, we consider positions of the 2 by 1 by 1 block which cross between two layers. Since each layer has 110 blocks (in 11 rows and 10 columns) then there are 110 positions for the 2 by 1 by 1 block between each pair of touching layers. Since there are $n - 1$ pairs of touching layers, then there are $110(n - 1)$ positions of this type.

Thus, overall we have 2362 total positions, so

$$199n + 110(n - 1) = 2362$$

$$309n - 110 = 2362$$

$$309n = 2472$$

$$n = 8$$

Answer: (B)







