



Canadian Mathematics Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 18, 2004

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iAnywhere Solutions

Time: 1 hour

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Calculators are permitted.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\frac{10}{10(11)-10^2}$ is
 (A) 100 (B) 2 (C) 10 (D) 1 (E) 11

2. $\sqrt{4^0 + 4^2 + 4^3}$ equals
 (A) 9 (B) 13 (C) 14 (D) 32 (E) 64

3. If $x = 2 - 4 + 6$ and $y = 1 - 3 + 5$, then $x - y$ equals
 (A) 0 (B) 1 (C) 5 (D) 3 (E) -1

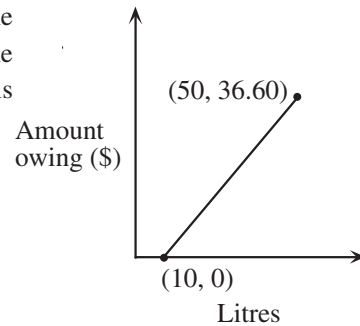
4. A lemon loaf completely fills a pan measuring 20 cm by 18 cm by 5 cm. The loaf is cut into 25 pieces of equal volume. If the density of the loaf is 2 g/cm^3 , how much does each of the 25 pieces weigh?
 (A) 72 g (B) 288 g (C) 36 g (D) 144 g (E) 720 g

5. If $\left(\frac{1}{2+3}\right)\left(\frac{1}{3+4}\right) = \frac{1}{x+5}$, the value of x is
 (A) 4 (B) 7 (C) 30 (D) 37 (E) 67

6. Three cans of juice fill $\frac{2}{3}$ of a one-litre jug. How many cans of juice are needed to completely fill 8 one-litre jugs?
 (A) 36 (B) 12 (C) $\frac{16}{3}$ (D) 16 (E) 24

7. When $x = \frac{1}{5}$, the value of the expression $\frac{x^2 - 4}{x^2 - 2x}$ is
 (A) 0.4 (B) -0.52 (C) -5 (D) 10 (E) 11



8. Jane arrives at the Fermat Fuel Fill-up to fill up her gas tank. The graph shows the amount of gas that Jane had upon arrival, the amount that she purchased, and the cost of this purchase. What is the price per litre of the gas that she purchased?
 (A) 91.5¢ (B) 73.2¢ (C) 61.0¢
 (D) 53.2¢ (E) \$1.09

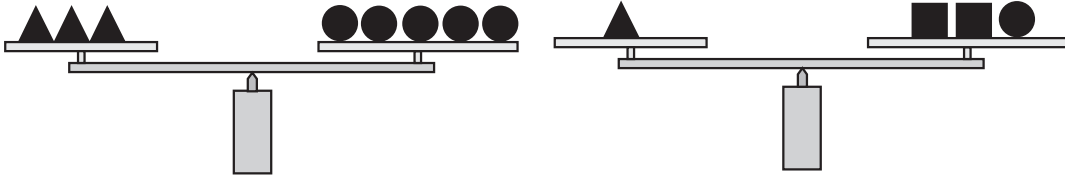


9. The table shows population information for two towns for the years 2003 and 2004.

Town	2003 Population	Percentage Change From 2003 to 2004
Cayleyville	10 000	4%
Pascalberg	25 000	-12%

What is the difference between the populations of the two towns in 2004?

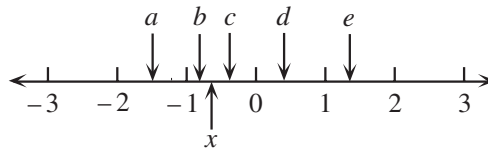
- (A) 12 400 (B) 11 600 (C) 17 600 (D) 13 800 (E) 17 400
10. In the diagram, two equal-armed balances are shown. How many  would it take to balance one ?



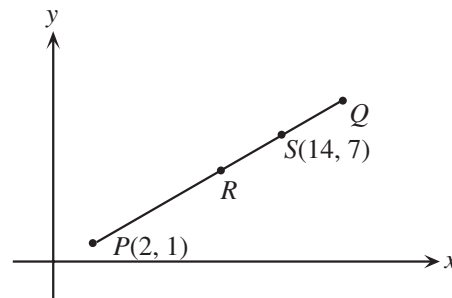
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Part B: Each correct answer is worth 6.

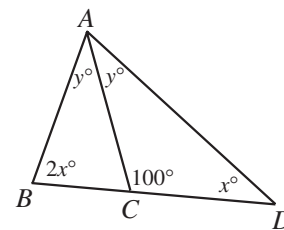
11. If x is located on the number line as shown, which letter best corresponds to the location of $-x^2$?



- (A) a (B) b (C) c (D) d (E) e
12. Point R is the midpoint of the line segment PQ and S is the midpoint of the line segment QR . If P has coordinates $(2, 1)$ and S has coordinates $(14, 7)$, then the coordinates of Q are



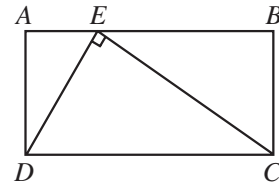
- (A) $(8, 4)$ (B) $(26, 13)$ (C) $(10, 5)$
 (D) $(18, 9)$ (E) $(16, 11)$
13. In the diagram, B, C and D lie on a straight line, with $\angle ACD = 100^\circ$, $\angle ADB = x^\circ$, $\angle ABD = 2x^\circ$, and $\angle DAC = \angle BAC = y^\circ$. The value of x is



- (A) 10 (B) 45 (C) 30
 (D) 50 (E) 20

14. In the diagram, $ABCD$ is a rectangle and point E lies on AB . Triangle DEC has $\angle DEC = 90^\circ$, $DE = 3$ and $EC = 4$. The length of AD is

(A) 2.6 (B) 2.4 (C) 2.8
 (D) 1.8 (E) 3.2

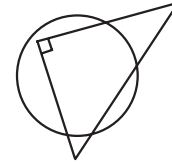


15. The graph of $x^2 - y^2 = 0$ is

(A) a straight line (B) a parabola (C) a circle
 (D) a single point (E) two straight lines

16. A right-angled triangle has sides of length 6, 8 and 10. A circle is drawn so that the area inside the circle but outside this triangle equals the area inside the triangle but outside the circle. The radius of the circle is closest to

(A) 2.9 (B) 2.8 (C) 3.8
 (D) 3.1 (E) 3.6



17. An increasing sequence is formed so that the difference between consecutive terms is a constant. If the first four terms of this sequence are x , y , $3x + y$, and $x + 2y + 2$, then the value of $y - x$ is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

18. If $y = a(x - 2)^2 + c$ and $y = (2x - 5)(x - b)$ represent the same quadratic function, the value of b is

(A) 3 (B) $\frac{3}{2}$ (C) $\frac{4}{5}$ (D) $-\frac{5}{2}$ (E) $\frac{8}{5}$

19. A computer software retailer has 1200 copies of a new software package to sell. From past experience, she knows that:

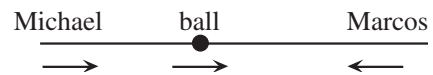
- Half of them will sell right away at the original price she sets,
- Two-thirds of the remainder will sell later when their price is reduced by 40%, and
- The remaining copies will sell in a clearance sale at 75% off the original price.

In order to make a reasonable profit, the total sales revenue must be \$72 000. To the nearest cent, what original price should she set?

(A) \$60.01 (B) \$75.01 (C) \$79.13 (D) \$80.90 (E) \$240.01

20. A soccer ball rolls at 4 m/s towards Marcos in a direct line from Michael. The ball is 15 m ahead of Michael who is chasing it at 9 m/s. Marcos is 30 m away from the ball and is running towards it at 8 m/s. The distance between Michael and Marcos when the ball is touched for the first time by one of them is closest to

(A) 2.00 m (B) 2.25 m (C) 2.50 m
 (D) 2.75 m (E) 3.00 m



Part C: Each correct answer is worth 8.

21. Bill and Jill are hired to paint a line on a road. If Bill works by himself, he could paint the line in B hours. If Jill works by herself, she could paint the line in J hours. Bill starts painting the line from one end, and Jill begins painting the line from the other end one hour later. They both work until the line is painted. Which of the following is an expression for the number of hours that Bill works?

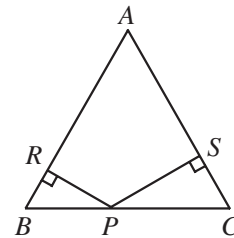
- (A) $\frac{B(J+1)}{B+J}$ (B) $J+1$ (C) $\frac{BJ}{B+J}+1$ (D) $\frac{B+J-1}{2}$ (E) $\frac{B(J-1)}{B+J}$

22. If k is the smallest positive integer such that $(2^k)(5^{300})$ has 303 digits when expanded, then the sum of the digits of the expanded number is

- (A) 11 (B) 10 (C) 8 (D) 7 (E) 5

23. Triangle ABC is isosceles with $AB = AC$ and $BC = 65$ cm. P is a point on BC such that the perpendicular distances from P to AB and AC are 24 cm and 36 cm, respectively. The area of $\triangle ABC$, in cm^2 , is

- (A) 1254 (B) 1640 (C) 1950
(D) 2535 (E) 2942

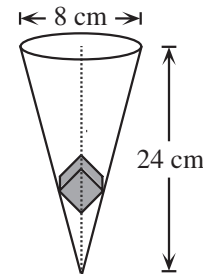


24. The polynomial $f(x)$ satisfies the equation $f(x) - f(x-2) = (2x-1)^2$ for all x . If p and q are the coefficients of x^2 and x , respectively, in $f(x)$, then $p+q$ is equal to

- (A) 0 (B) $\frac{5}{6}$ (C) $\frac{4}{3}$ (D) 1 (E) $\frac{2}{3}$

25. A steel cube has edges of length 3 cm, and a cone has a diameter of 8 cm and a height of 24 cm. The cube is placed in the cone so that one of its interior diagonals coincides with the axis of the cone. What is the distance, in cm, between the vertex of the cone and the closest vertex of the cube?

- (A) $6\sqrt{6} - \sqrt{3}$ (B) $\frac{12\sqrt{6} - 3\sqrt{3}}{4}$ (C) $6\sqrt{6} - 2\sqrt{3}$
(D) $5\sqrt{3}$ (E) $6\sqrt{6}$



PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions

Copies of previous contests and solutions are available at no cost in both English and French at <http://www.cemc.uwaterloo.ca>

Problems Problems Problems Books

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is \$15. **(Available in English only.)**

Volume 1

- over 300 problems and full solutions
- 10 topics
- for students in Grades 9, 10, & 11
- French version of Volume 1 is available

Volume 3

- over 235 problems and full solutions
- 12 topics
- for senior high school students

Volume 5

- over 200 problems and full solutions
- 9 topics (different from Volume 3)
- for senior high school students

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- over 300 problems and full solutions
- 12 topics
- for students in Grades 9 and 10

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- over 300 problems and full solutions
- 11 topics
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- 10 topics (different from Volume 1)
- for students in Grades 9, 10, & 11

Volume 4

- over 325 problems and full solutions
- 12 topics
- for students in Grades 7, 8, & 9

Volume 6

- over 300 problems and full solutions
- 11 topics
- for students in Grades 7, 8, & 9

Volume 8

- over 200 problems and full solutions
- 10 topics
- for students in Grades 11 and 12

Orders should be addressed to: Canadian Mathematics Competition
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University of Waterloo
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Include your name, address (with postal code), and telephone number.

Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add \$3.00 for the first item ordered for shipping and handling, plus \$1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders *outside of Canada ONLY*, add \$10.00 for the first item ordered for shipping and handling, plus \$2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2004.**

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