Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest *(Grade 10)*

Wednesday, February 18, 2004

Time: 1 hour

Calculators are permitted.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $2^2+1^2+0^2+(-1)^2+(-2)^2$ is

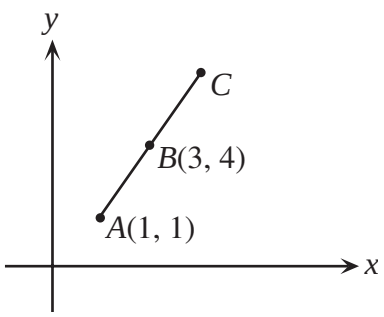
(**A**) 5 (**B**) -6 (**C**) 3 (**D**) 6 (**E**) 10

2. 25% of 2004 is equal to

(**A**) 50% of 4008 (**B**) 50% of 1002 (**C**) 100% of 1002
(**D**) 10% of 8016 (**E**) 20% of 3006

3. Point $B(3,4)$ is the midpoint of the line segment joining the points $A(1,1)$ and C. The coordinates of C are

(**A**) $(2,3)$ (**B**) $(2,2)$ (**C**) $(4,6)$
(**D**) $(5,8)$ (**E**) $(5,7)$



4. If $x+1-2+3-4=5-6+7-8$, the value of x is

(**A**) -2 (**B**) -1 (**C**) 0 (**D**) 1 (**E**) 2

5. In the sequence, each figure is made up of small squares of side length 1. What is the outer perimeter of the fifth figure in the sequence?

(**A**) 9 (**B**) 18 (**C**) 20 (**D**) 24 (**E**) 36

6. If $x+6y=17$, the value of $7x+42y$ is

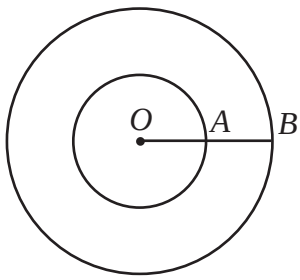
(**A**) 24 (**B**) 42 (**C**) 49 (**D**) 102 (**E**) 119

7. If $3^2+3^2+3^2=3^a$, the value of a is

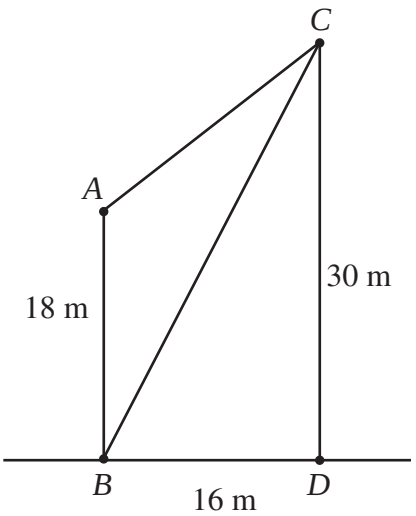
(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 6 (**E**) 8

8. In the diagram, O is the centre of each circle. The circumferences of the circles are 24π and 14π. B is a point on the outer circle and OB intersects the inner circle at A. The length of AB is

(**A**) $\sqrt{10}$ (**B**) 5 (**C**) 7
(**D**) 10π (**E**) 3

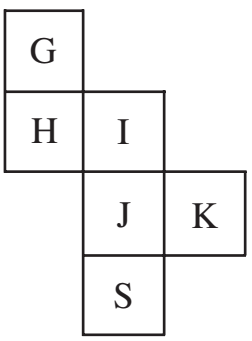


9. Two vertical towers, AB and CD, are located 16 m apart on flat ground, as shown. Tower AB is 18 m tall and tower CD is 30 m tall. Ropes are tied from A to C and from B to C. Assuming the ropes are taut, the total length of rope, in m, is



(**A**) 54 (**B**) 64 (**C**) 44
(**D**) 48 (**E**) 59

10. If the figure shown is folded to make a cube, what letter is opposite G?



(**A**) S (**B**) H (**C**) I
(**D**) J (**E**) K

Part B: Each correct answer is worth 6.

11. In the sequence of five numbers x, _____, 3, _____, 18, each number after the second is obtained by multiplying the two previous numbers. The value of x is

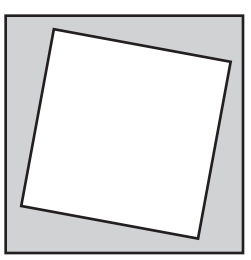
(**A**) $\frac{2}{3}$ (**B**) $\frac{3}{2}$ (**C**) 1 (**D**) -9 (**E**) -1

12. In the magic square, the sum of the three numbers in any row, column or diagonal is the same. The sum of the three numbers in any row is

$2x$	3	2
		-3
0	x	

(**A**) 0 (**B**) 1 (**C**) 3
(**D**) 7 (**E**) 9

13. In the diagram, a smaller square lies inside a larger square. The perimeter of the smaller square is 72 cm. The shaded area is 160 cm^2. The perimeter of the larger square, in cm, is

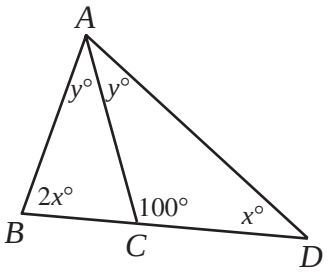


(**A**) 58 (**B**) 88 (**C**) 116
(**D**) 121 (**E**) 112

14. If x and y are positive numbers and the average of 4, 20 and x is equal to the average of y and 16, then the ratio $x:y$ is

(**A**) 3:2 (**B**) 2:3 (**C**) 1:1 (**D**) 2:5 (**E**) 5:2

15. In the diagram, B, C and D lie on a straight line, with $\angle ACD = 100^\circ$, $\angle ADB = x^\circ$, $\angle ABD = 2x^\circ$, and $\angle DAC = \angle BAC = y^\circ$. The value of x is



(**A**) 10 (**B**) 45 (**C**) 30
(**D**) 50 (**E**) 20

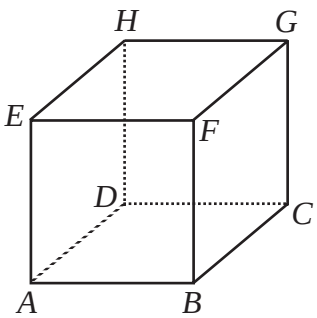
16. In a dice game, a player rolls two dice. His score is the larger of the two numbers on the dice. For example, if he rolls 3 and 5, his score is 5, and if he rolls 4 and 4, his score is 4. What is the probability that his score is 3 or less?

(**A**) $\frac{1}{4}$ (**B**) $\frac{7}{36}$ (**C**) $\frac{5}{36}$ (**D**) $\frac{1}{3}$ (**E**) $\frac{2}{9}$

17. The two whole numbers m and n satisfy $m+n=20$ and $\frac{1}{m}+\frac{1}{n}=\frac{5}{24}$. The product mn is equal to

(**A**) 72 (**B**) 36 (**C**) 48 (**D**) 96 (**E**) 24

18. In the diagram, $ABCDEFGH$ is a cube with an edge length of 12 cm. An ant sits on the cube at vertex A. The ant can only walk along the edges of the cube, and cannot walk along any edge more than once. What is the greatest distance that the ant can walk before it cannot continue?

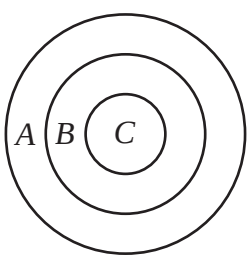


(**A**) 96 cm (**B**) 144 cm (**C**) 84 cm
(**D**) 108 cm (**E**) 132 cm

19. $\frac{1}{2}+\frac{2^1}{2^2}+\frac{2^2}{2^3}+\cdots+\frac{2^{2002}}{2^{2003}}+\frac{2^{2003}}{2^{2004}}$ is equal to

(**A**) 1002 (**B**) 501 (**C**) $\frac{1}{2^{2004}}$ (**D**) 2004 (**E**) $\frac{2004}{2^{2004}}$

20. An archery target has 3 regions, each worth a different value if it is hit. Three archers shoot two arrows each and record scores as follows:



First archer: 1 arrow in C and 1 arrow in A for a score of 15 points
Second archer: 1 arrow in C and 1 arrow in B for a score of 18 points
Third archer: 1 arrow in B and 1 arrow in A for a score of 13 points

If a fourth archer shoots 2 arrows into ring B, her score is

(**A**) 10 (**B**) 14 (**C**) 16 (**D**) 18 (**E**) 20

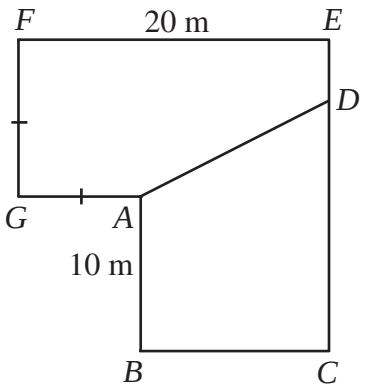
continued ...

Part C: Each correct answer is worth 8.

21. In a pack of construction paper, the numbers of blue and red sheets are originally in the ratio 2:7. Each day, Laura uses 1 blue sheet and 3 red sheets. One day, she uses 3 red sheets and the last blue sheet, leaving her with 15 red sheets. How many sheets of construction paper were in the pack originally?

(**A**) 144 (**B**) 252 (**C**) 135 (**D**) 270 (**E**) 105

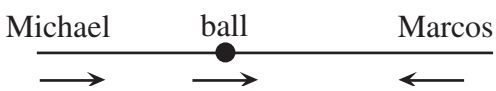
22. In the diagram, $ABCDEFG$ is a room having square corners, with $EF = 20$ m, $AB = 10$ m, and $AG = GF$. The total area of the room is $280\ \text{m}^2$. A wall is built from A to D creating two rooms of equal area. What is the distance, in metres, from C to D?



(**A**) 15 (**B**) $\frac{50}{3}$ (**C**) 12

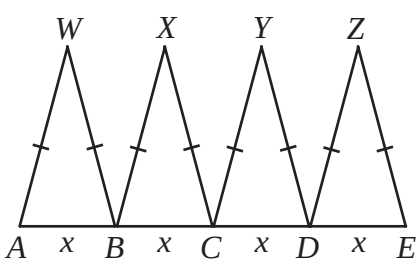
(**D**) 13 (**E**) $\frac{40}{3}$

23. A soccer ball rolls at 4 m/s towards Marcos in a direct line from Michael. The ball is 15 m ahead of Michael who is chasing it at 9 m/s. Marcos is 30 m away from the ball and is running towards it at 8 m/s. The distance between Michael and Marcos when the ball is touched for the first time by one of them is closest to



(**A**) 2.00 m (**B**) 2.25 m (**C**) 2.50 m
(**D**) 2.75 m (**E**) 3.00 m

24. Four identical isosceles triangles AWB, BXC, CYD, and DZE are arranged, as shown, with points A, B, C, D, and E lying on the same straight line. A new triangle is formed with sides the same lengths as AX, AY and AZ. If $AZ = AE$, the largest integer value of x such that the area of this new triangle is less than 2004 is



(**A**) 18 (**B**) 19 (**C**) 20
(**D**) 21 (**E**) 22

25. The number of positive integers x with $x \le 60$ such that each of the rational expressions

$$\frac{7x+1}{2}, \frac{7x+2}{3}, \frac{7x+3}{4}, \ldots, \frac{7x+300}{301}$$

is in lowest terms (i.e. in each expression, the numerator and denominator have no common factors) is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions

Copies of previous contests and solutions are available at no cost in both English and French at http://www.cemc.uwaterloo.ca

Problems Problems Problems Books

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is $15. **(Available in English only.)**

Volume 1
- over 300 problems and full solutions
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- French version of Volume 1 is available

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- 12 topics
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