



# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## *Cayley Contest* (Grade 10)

Wednesday, February 20, 2002

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**Time:** 1 hour

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**Calculators are permitted**, providing they are non-programmable and without graphic displays.

### Instructions

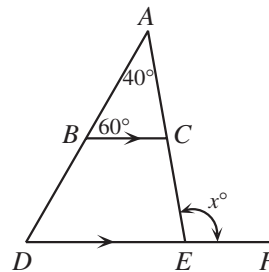
1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

- $5x + 2(4 + x)$  is equal to  
(A)  $5x + 8$       (B)  $3x + 6$       (C)  $7x + 8$       (D)  $7x + 6$       (E)  $6x + 8$
- The value of  $(2 + 3)^2 - (2^2 + 3^2)$  is  
(A) 12      (B) 0      (C) 30      (D) 16      (E) -3
- If  $x = -3$ , the numerical value of  $x^2 - 4(x - 5)$  is  
(A) 40      (B) 38      (C) -23      (D) 41      (E) -26
- If  $n$  is  $\frac{5}{6}$  of 240, then  $\frac{2}{5}$  of  $n$  is  
(A) 288      (B) 80      (C) 96      (D) 200      (E) 500
- The numerical value of  $2^{-2} \times 2^{-1} \times 2^0 \times 2^1 \times 2^2$  is  
(A) 4      (B) 1      (C) 0      (D)  $\frac{1}{4}$       (E)  $\frac{1}{2}$

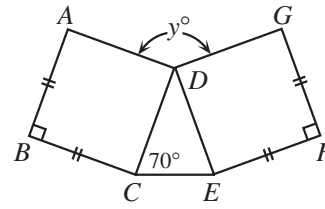
- In the diagram, the value of  $x$  is  
(A) 130      (B) 120      (C) 110  
(D) 100      (E) 80



- If the point  $(-2, 4)$  is on a line with slope  $\frac{1}{2}$ , then the y-intercept of this line is  
(A) 5      (B) -4      (C) 3      (D) 0      (E) 8
- After having played three basketball games, Megan had scored an average of 18 points per game. After her fourth game, her scoring average had dropped to 17 points per game. How many points did Megan score in her fourth game?  
(A) 18      (B) 17      (C) 16      (D) 15      (E) 14

9. In the diagram,  $ABCD$  and  $DEFG$  are squares with equal side lengths, and  $\angle DCE = 70^\circ$ . The value of  $y$  is

(A) 120            (B) 160            (C) 130  
 (D) 110            (E) 140



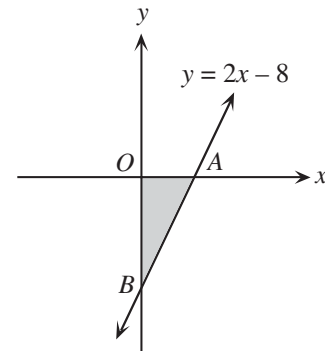
10. Faruq subtracted 5 from a number and then divided by 4. Next, he subtracted 4 from the original number and then divided by 5. He got the same final answer both times. The original number was

(A) 4            (B) 15            (C) 9            (D) 20            (E) -9

**Part B: Each correct answer is worth 6.**

11. In the diagram, the line with equation  $y = 2x - 8$  crosses the  $x$ -axis at  $A$  and the  $y$ -axis at  $B$ . The area of  $\triangle AOB$  is

(A) 8            (B) 16            (C) 12  
 (D) 32            (E) 4

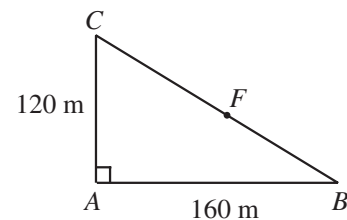


12. A compact disc originally sells for \$10.00. If the price of the compact disc is increased by 40% and this new price is later decreased by 30%, what is the final price?

(A) \$9.80            (B) \$17.00            (C) \$9.00            (D) \$19.80            (E) \$9.60

13. In the diagram,  $ABC$  represents a triangular jogging path. Jack jogs along the path from  $A$  to  $B$  to  $F$ . Jill jogs from  $A$  to  $C$  to  $F$ . Each jogs the same distance. The distance from  $F$  to  $B$ , in metres, is

(A) 40            (B) 120            (C) 100  
 (D) 80            (E) 200

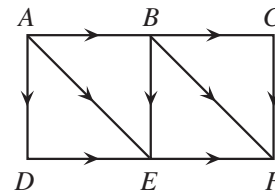


14. If  $a(c+d) + b(c+d) = 42$  and  $c+d = 3$ , what is the value of  $a+b+c+d$ ?

(A) 14            (B) 56            (C) 3            (D) 17            (E) 39

15. In the grid shown, it is only possible to travel along an edge in the direction indicated by the arrow. The number of different paths from  $A$  to  $F$  is

(A) 9            (B) 5            (C) 3  
 (D) 6            (E) 4



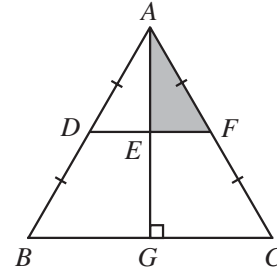
16. If the product of four consecutive positive integers is 358 800, then the sum of these four integers is

(A) 102            (B) 98            (C) 94            (D) 90            (E) 106

17. A “double-single” number is a three-digit number made up of two identical digits followed by a different digit. For example, 553 is a double-single number. How many double-single numbers are there between 100 and 1000?

(A) 81                      (B) 18                      (C) 72                      (D) 64                      (E) 90

18. In the diagram, triangle  $ABC$  is isosceles with  $AB = AC$ , and  $AG$  is perpendicular to  $BC$ . Point  $D$  is the midpoint of  $AB$ , point  $F$  is the midpoint of  $AC$ , and  $E$  is the point of intersection of  $DF$  and  $AG$ . What fraction of the area of  $\triangle ABC$  does the shaded area represent?



(A)  $\frac{1}{12}$                       (B)  $\frac{1}{6}$                       (C)  $\frac{1}{4}$   
 (D)  $\frac{1}{10}$                       (E)  $\frac{1}{8}$

19. The sum of the digits of the integer equal to  $777\,777\,777\,777\,777^2 - 222\,222\,222\,222\,223^2$  is

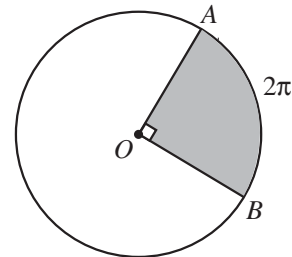
(A) 148                      (B) 84                      (C) 74                      (D) 69                      (E) 79

20. Two cylindrical tanks sit side by side on a level surface. The first tank has a radius of 4 metres, a height of 10 metres, and is full of water. The second tank has a radius of 6 metres, a height of 8 metres, and is empty. Water is pumped from the first tank to the second until the depth of water in both tanks is the same. The depth of water in each tank, in metres, is

(A) 4                      (B) 5                      (C)  $\frac{46}{15}$                       (D)  $\frac{52}{17}$                       (E)  $\frac{40}{13}$

**Part C: Each correct answer is worth 8.**

21. In the diagram, the circle has centre  $O$ . The shaded sector  $AOB$  has sector angle  $90^\circ$ , and  $AB$  has arc length  $2\pi$  units. The area of sector  $AOB$  is

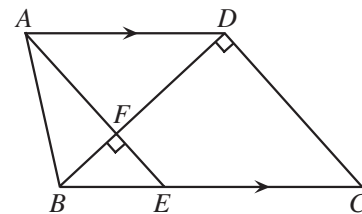


(A)  $4\pi$                       (B)  $16\pi$                       (C)  $6\pi$   
 (D)  $24\pi$                       (E)  $8\pi$

22. In how many ways can 75 be expressed as the sum of two or more consecutive positive integers?

(A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

23. In trapezoid  $ABCD$ ,  $AD$  is parallel to  $BC$ . Also,  $BD$  is perpendicular to  $DC$ . The point  $F$  is chosen on line  $BD$  so that  $AF$  is perpendicular to  $BD$ .  $AF$  is extended to meet  $BC$  at point  $E$ . If  $AB = 41$ ,  $AD = 50$  and  $BF = 9$ , what is the area of quadrilateral  $FECD$ ?

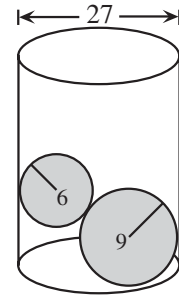


(A) 900                      (B) 1523.5                      (C) 960  
 (D) 1560                      (E) 1300

continued ...

24. A cylinder, which has a diameter of 27 and a height of 30, contains two lead spheres with radii 6 and 9, with the larger sphere sitting on the bottom of the cylinder, as shown. Water is poured into the cylinder so that it just covers both spheres. The volume of water required is

(A)  $3672\pi$       (B)  $3660\pi$       (C)  $3375\pi$   
(D)  $3114\pi$       (E)  $4374\pi$



25. A *lattice point* is a point  $(x, y)$  where both  $x$  and  $y$  are integers. For how many different integer values of  $k$  will the two lines  $kx - 5y + 7 = 0$  and  $k^2x - 5y + 1 = 0$  intersect at a lattice point?

(A) 1      (B) 2      (C) 3      (D) 4      (E) 5