Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Euclid Contest (Grade 12)

for

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

Awards

Thursday, April 19, 2001

Time: $2\frac{1}{2}$ hours

Calculators are permitted, provided they are non-programmable and without graphic displays.

Do not open this booklet until instructed to do so. The paper consists of 10 questions, each worth 10 marks. Parts of each question can be of two types. **SHORT ANSWER** parts are worth 2 marks each (questions 1-2) or 3 marks each (questions 3-7). **FULL SOLUTION** parts are worth the remainder of the 10 marks for the question.

Instructions for SHORT ANSWER parts:

1. **SHORT ANSWER** parts are indicated like this: .
2. **Enter the answer in the appropriate box in the answer booklet.** For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

Instructions for FULL SOLUTION parts:

1. **FULL SOLUTION** parts are indicated like this: .
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, foolscap will be supplied by your supervising teacher. Insert these pages into your answer booklet.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution poorly presented will not earn full marks.

NOTE: At the completion of the contest, insert the information sheet inside the answer booklet.

NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Place all answers in the answer booklet provided.
3. For questions marked " ", full marks will be given for a correct answer placed in the appropriate box in the answer booklet. **Marks may be given for work shown**. Students are strongly encouraged to show their work.
4. It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2+\sqrt{7}$, etc., except where otherwise indicated.

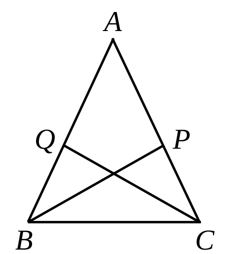
1. (a) What are the values of x such that $(2x-3)^2 = 9$?

(b) If $f(x) = x^2 - 3x - 5$, what are the values of k such that $f(k) = k$?

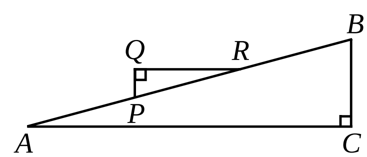
(c) Determine all (x, y) such that $x^2 + y^2 = 25$ and $x - y = 1$.

2. (a) The vertex of the parabola $y = (x-b)^2 + b + h$ has coordinates $(2, 5)$. What is the value of h?

(b) In the isosceles triangle ABC, $AB = AC$ and $\angle BAC = 40°$. Point P is on AC such that BP is the bisector of $\angle ABC$. Similarly, Q is on AB such that CQ bisects $\angle ACB$. What is the size of $\angle APB$, in degrees?



(c) In the diagram, $AB = 300$, $PQ = 20$, and $QR = 100$. Also, QR is parallel to AC. Determine the length of BC, to the nearest integer.

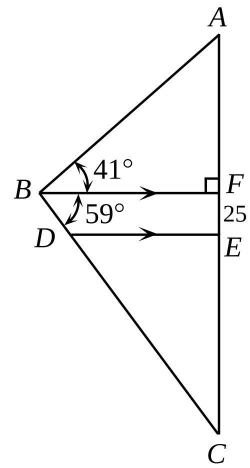


3. (a) In an increasing sequence of numbers with an odd number of terms, the difference between any two consecutive terms is a constant d, and the middle term is 302. When the last 4 terms are removed from the sequence, the middle term of the resulting sequence is 296. What is the value of d?

(b) There are two increasing sequences of five consecutive integers, each of which have the property that the sum of the squares of the first three integers in the sequence equals the sum of the squares of the last two. Determine these two sequences.

4. (a) If $f(t)=\sin\left(\pi t-\frac{\pi}{2}\right)$, what is the smallest positive value of t at which $f(t)$ attains its minimum value?

(b) In the diagram, $\angle ABF=41°$, $\angle CBF=59°$, DE is parallel to BF, and $EF=25$. If $AE=EC$, determine the length of AE, to 2 decimal places.



5. (a) Determine all integer values of x such that $\left(x^2-3\right)\left(x^2+5\right)<0$.

(b) At present, the sum of the ages of a husband and wife, P, is six times the sum of the ages of their children, C. Two years ago, the sum of the ages of the husband and wife was ten times the sum of the ages of the same children. Six years from now, it will be three times the sum of the ages of the same children. Determine the number of children.

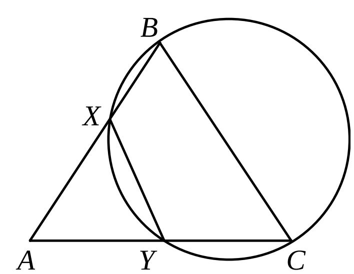
6. (a) Four teams, A, B, C, and D, competed in a field hockey tournament. Three coaches predicted who would win the Gold, Silver and Bronze medals:

Medal	Gold	Silver	Bronze
Team			

- Coach 1 predicted Gold for A, Silver for B, and Bronze for C,
- Coach 2 predicted Gold for B, Silver for C, and Bronze for D,
- Coach 3 predicted Gold for C, Silver for A, and Bronze for D.

Each coach predicted exactly one medal winner correctly. Complete the table **in the answer booklet** to show which team won which medal.

(b) In triangle ABC, $AB=BC=25$ and $AC=30$. The circle with diameter BC intersects AB at X and AC at Y. Determine the length of XY.



7. (a) What is the value of x such that $\log_2(\log_2(2x-2))=2$?

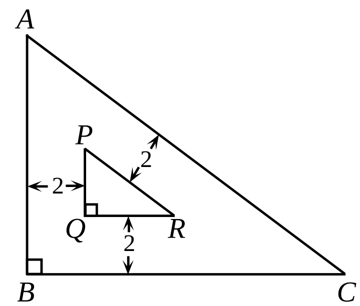
(b) Let $f(x)=2^{kx}+9$, where k is a real number. If $f(3):f(6)=1:3$, determine the value of $f(9)-f(3)$.

8. (a) On the grid provided in the answer booklet, sketch $y = x^2 - 4$ and $y = 2|x|$.

(b) Determine, with justification, all values of k for which $y = x^2 - 4$ and $y = 2|x| + k$ do **not** intersect.

(c) State the values of k for which $y = x^2 - 4$ and $y = 2|x| + k$ intersect in exactly two points. (Justification is not required.)

9. Triangle ABC is right-angled at B and has side lengths which are integers. A second triangle, PQR, is located inside ΔABC as shown, such that its sides are parallel to the sides of ΔABC and the distance between parallel lines is 2. Determine the side lengths of all possible triangles ABC, such that the area of ΔABC is 9 times that of ΔPQR.



10. Points P and Q are located inside the square $ABCD$ such that DP is parallel to QB and $DP = QB = PQ$. Determine the minimum possible value of $\angle ADP$.