



Canadian Mathematics Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 21, 2001

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Time: 1 hour

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Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 20.

Part A: Each correct answer is worth 5.

1. The value of $\frac{5(6)-3(4)}{6+3}$ is

- (A) 1 (B) 2 (C) 6 (D) 12 (E) 31

2. When $\frac{1}{4}$ of 15 is multiplied by $\frac{1}{3}$ of 10, the answer is

- (A) 5 (B) $\frac{25}{2}$ (C) $\frac{85}{12}$ (D) $\frac{99}{8}$ (E) $\frac{25}{7}$

3. If $x = \frac{1}{4}$, which of the following has the largest value?

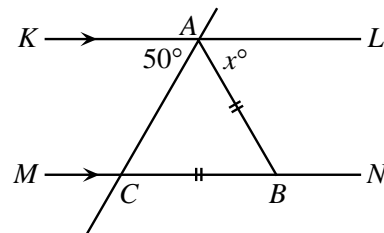
- (A) x (B) x^2 (C) $\frac{1}{2}x$ (D) $\frac{1}{x}$ (E) \sqrt{x}

4. In a school, 30 boys and 20 girls entered the Cayley competition. Certificates were awarded to 10% of the boys and 20% of the girls. Of the students who participated, the percentage that received certificates was

- (A) 14 (B) 15 (C) 16 (D) 30 (E) 50

5. In the diagram, KL is parallel to MN , $AB = BC$, and $\angle KAC = 50^\circ$. The value of x is

- (A) 40 (B) 65 (C) 25
(D) 100 (E) 80

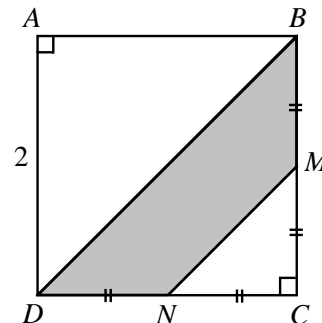


6. Dean scored a total of 252 points in 28 basketball games. Ruth played 10 fewer games than Dean. Her scoring average was 0.5 points per game higher than Dean's scoring average. How many points, in total, did Ruth score?

- (A) 153 (B) 171 (C) 180 (D) 266 (E) 144

7. In the diagram, square $ABCD$ has side length 2, with M the midpoint of BC and N the midpoint of CD . The area of the shaded region $BMND$ is

- (A) 1 (B) $2\sqrt{2}$ (C) $\frac{4}{3}$
(D) $\frac{3}{2}$ (E) $4 - \frac{3}{2}\sqrt{2}$

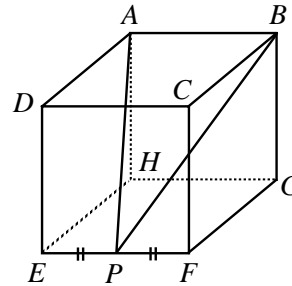


16. How many integer values of x satisfy $\frac{x-1}{3} < \frac{5}{7} < \frac{x+4}{5}$?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

17. $ABCDEFGH$ is a cube having a side length of 2. P is the midpoint of EF , as shown. The area of $\triangle APB$ is

- (A) $\sqrt{8}$ (B) 3 (C) $\sqrt{32}$
 (D) $\sqrt{2}$ (E) 6



18. How many five-digit positive integers, divisible by 9, can be written using only the digits 3 and 6?

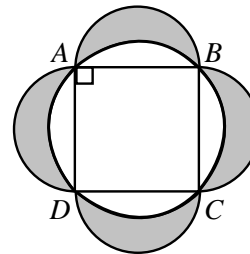
- (A) 5 (B) 2 (C) 12 (D) 10 (E) 8

19. Three different numbers are chosen such that when each of the numbers is added to the average of the remaining two, the numbers 65, 69 and 76 result. The average of the three original numbers is

- (A) 34 (B) 35 (C) 36 (D) 37 (E) 38

20. Square $ABCD$ with side length 2 is inscribed in a circle, as shown. Using each side of the square as a diameter, semi-circular arcs are drawn. The area of the shaded region outside the circle and inside the semi-circles is

- (A) π (B) 4 (C) $2\pi - 2$
 (D) $\pi + 1$ (E) $2\pi - 4$



Part C: Each correct answer is worth 8.

21. Point P is on the line $y = 5x + 3$. The coordinates of point Q are $(3, -2)$. If M is the midpoint of PQ , then M must lie on the line

- (A) $y = \frac{5}{2}x - \frac{7}{2}$ (B) $y = 5x + 1$ (C) $y = -\frac{1}{5}x - \frac{7}{5}$ (D) $y = \frac{5}{2}x + \frac{1}{2}$ (E) $y = 5x - 7$

22. What is the shortest distance between two circles, the first having centre $A(5, 3)$ and radius 12, and the other with centre $B(2, -1)$ and radius 6?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

23. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C. What is the total height, in cm, of the bottle?

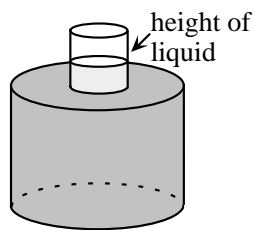


Figure A

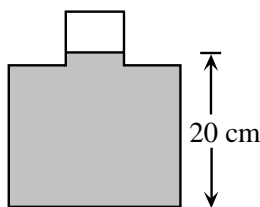


Figure B

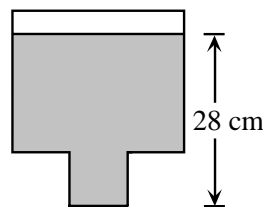


Figure C

- (A) 29 (B) 30 (C) 31 (D) 32 (E) 48
24. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2882 is a four-digit palindrome and 49194 is a five-digit palindrome. There are pairs of four-digit palindromes whose sum is a five-digit palindrome. One such pair is 2882 and 9339. How many such pairs are there?

- (A) 28 (B) 32 (C) 36 (D) 40 (E) 44

25. The circle with centre A has radius 3 and is tangent to both the positive x -axis and positive y -axis, as shown. Also, the circle with centre B has radius 1 and is tangent to both the positive x -axis and the circle with centre A . The line L is tangent to both circles. The y -intercept of line L is

- (A) $3 + 6\sqrt{3}$ (B) $10 + 3\sqrt{2}$ (C) $8\sqrt{3}$
 (D) $10 + 2\sqrt{3}$ (E) $9 + 3\sqrt{3}$

