35th Anniversary 1963 – 1998
Canadian Mathematics Competition
An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

1998 Solutions

Gauss Contest

(Grade 8)

Part A

1. The number 4567 is tripled. The ones digit (units digit) in the resulting number is
   (**A**) 5 (**B**) 6 (**C**) 7 (**D**) 3 (**E**) 1

   Solution
   If we wish to determine the units digit when we triple 4567, it is only necessary to triple 7 and take the units digit of the number 21.
   The required number is 1. ANSWER: (E)

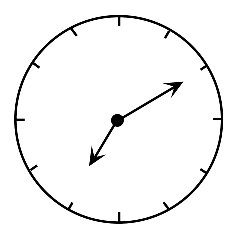
2. The smallest number in the set $\{0, -17, 4, 3, -2\}$ is
   (**A**) -17 (**B**) 4 (**C**) -2 (**D**) 0 (**E**) 3

   Solution
   By inspection, the smallest number is -17. ANSWER: (A)

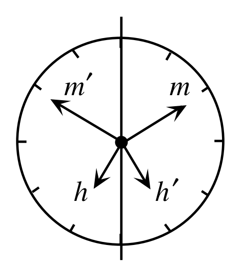
3. The average of -5, -2, 0, 4, and 8 is
   (**A**) $\frac{5}{4}$ (**B**) 0 (**C**) $\frac{19}{5}$ (**D**) 1 (**E**) $\frac{9}{4}$

   Solution
   The sum of the integers is 5.
   They have an average of 1. ANSWER: (D)

4. Emily sits on a chair in a room. Behind her is a clock. In front of her is a mirror. In the mirror, she sees the image of the clock as shown. The actual time is closest to
   (**A**) 4:10 (**B**) 7:10 (**C**) 5:10
   (**D**) 6:50 (**E**) 4:50



   Solution
   Draw a mirror line to run from 12 o'clock to 6 o'clock and then reflect both the minute and hour hand in this mirror line. The minute hand (m) is reflected to 10 o'clock (which is labelled m') and the hour hand (h) is reflected to just before 5 o'clock (h').
   The required time is 4:50.



ANSWER: (E)

5. If 1.2×10^6 is doubled, what is the result?

(**A**) 2.4×10^6 (**B**) 2.4×10^{12} (**C**) 2.4×10^3 (**D**) 1.2×10^{12} (**E**) 0.6×10^{12}

Solution

Doubling the given number means that 1.2 must be doubled.

The required number is 2.4×10^6. ANSWER: (A)

6. Tuesday's high temperature was 4°C warmer than that of Monday's. Wednesday's high temperature was 6°C cooler than that of Monday's. If Tuesday's high temperature was 22°C, what was Wednesday's high temperature?

(**A**) 20°C (**B**) 24°C (**C**) 12°C (**D**) 32°C (**E**) 16°C

Solution

If Tuesday's temperature was 22°C then Monday's high temperature was 18°C.

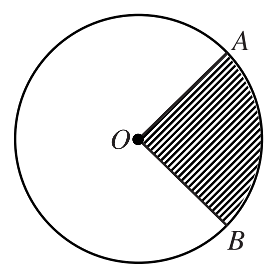
Wednesday's temperature was 12°C since it was 6°C cooler than that of Monday's high temperature.

ANSWER: (C)

7. In the circle with centre *O*, the shaded sector represents 20% of the area of the circle. What is the size of angle *AOB*?

(**A**) 36° (**B**) 72° (**C**) 90°

(**D**) 80° (**E**) 70°



Solution

If the area of the sector represents 20% of the area of the circle then angle *AOB* is 20% of 360° or 72°.

ANSWER: (B)

8. The pattern of figures △●□▲○ is repeated in the sequence

△, ●, □, ▲, ○, △, ●, □, ▲, ○, ...

The 214th figure in the sequence is

(**A**) △ (**B**) ● (**C**) □ (**D**) ▲ (**E**) ○

Solution

Since the pattern repeats itself after every five figures, it begins again after 210 figures have been completed.

The 214th figure would be the fourth element in the sequence or ▲. ANSWER: (D)

9. When a pitcher is $\frac{1}{2}$ full it contains exactly enough water to fill three identical glasses. How full would the pitcher be if it had exactly enough water to fill four of the same glasses?

(A) $\frac{2}{3}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{4}{7}$ **(D)** $\frac{6}{7}$ **(E)** $\frac{3}{4}$

Solution

If three glasses of water are the same as $\frac{1}{2}$ a pitcher then one glass is the same as $\frac{1}{6}$ of the pitcher.

If there were four glasses of water in the pitcher, this would be the same as $\frac{4}{6} = \frac{2}{3}$ of a pitcher.

ANSWER: (A)

10. A bank employee is filling an empty cash machine with bundles of \$5.00, \$10.00 and \$20.00 bills. Each bundle has 100 bills in it and the machine holds 10 bundles of each type. What amount of money is required to fill the machine?

(A) \$30 000 **(B)** \$25 000 **(C)** \$35 000 **(D)** \$40 000 **(E)** \$45 000

Solution

Since there are three bundles, each with 100 bills in them, the three bundles would be worth \$500, \$1000 and \$2000 respectively.

Since there are 10 bundles of each type of bill, their overall value would be

$10(\$500 + \$1000 + \$2000) = \$35\,000$. ANSWER: (C)

Part B

11. The weight limit for an elevator is 1500 kilograms. The average weight of the people in the elevator is 80 kilograms. If the combined weight of the people is 100 kilograms over the limit, how many people are in the elevator?

(A) 14 **(B)** 17 **(C)** 16 **(D)** 20 **(E)** 13

Solution

The combined weight of the people on the elevator is 100 kilograms over the limit which implies that their total weight is 1600 kilograms.

If the average weight is 80 kilograms there must be $\frac{1600}{80}$ or 20 people on the elevator.

ANSWER: (D)

12. In the 4×4 square shown, each row, column and diagonal should contain each of the numbers 1, 2, 3, and 4. Find the value of $K + N$.

(A) 4 **(B)** 3 **(C)** 5

(D) 6 **(E)** 7

1	*F*	*G*	*H*
T	2	*J*	*K*
L	*M*	3	*N*
P	*Q*	1	*R*

Solution

Since R is on a main diagonal and the numbers 1, 2 and 3 have already been used on this diagonal, then $R = 4$.
The easiest way to look at how to arrange the numbers is to look at boxes P and Q.
Q must be either 2 or 3 but since there is already a 2 in the same column as Q, we conclude that $Q = 3$ and $P = 2$.

1			
	2		
		3	
2	3	1	4

'cannot be 2'

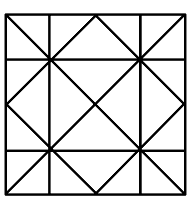
From this point we simply fill in the boxes according to the rule that each row, column and diagonal contains each of the numbers 1, 2, 3, and 4.
Doing this, we arrive at the following arrangement of numbers.

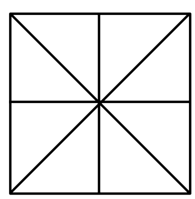
1	4	2	3
3	2	4	1
4	1	3	2
2	3	1	4

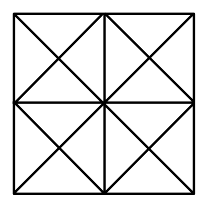
We see that $K + N = 3$.
NOTE 1: It is not necessary that we complete all the boxes but it is a useful way to verify the overall correctness of our work.
NOTE 2: We could have started by considering H, K and N but this takes a little longer to complete.

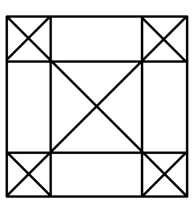
ANSWER: (B)

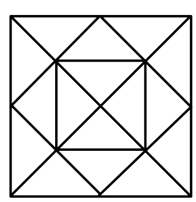
13. Claire takes a square piece of paper and folds it in half four times without unfolding, making an isosceles right triangle each time. After unfolding the paper to form a square again, the creases on the paper would look like

(**A**) 

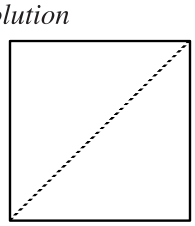
(**B**) 

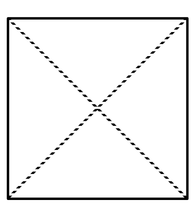
(**C**) 

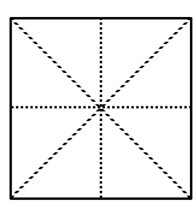
(**D**) 

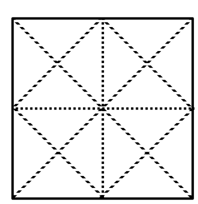
(**E**) 

Solution


Fold 1


Fold 2


Fold 3


Fold 4

ANSWER: (C)

14. Stephen had a 10:00 a.m. appointment 60 km from his home. He averaged 80 km/h for the trip and arrived 20 minutes late for the appointment. At what time did he leave his home?
(**A**) 9:35 a.m. (**B**) 9:15 a.m. (**C**) 8:40 a.m. (**D**) 9:00 a.m. (**E**) 9:20 a.m.

Solution
If Stephen averaged 80 km/h for a trip which was 60 km in length, it must have taken him 45 minutes to make the trip.
If the entire trip took 45 minutes and he arrived 20 minutes late he must have left home at 9:35 a.m.
ANSWER: (A)

15. Michael picks three *different* digits from the set $\{1, 2, 3, 4, 5\}$ and forms a mixed number by placing the digits in the spaces of $\square\frac{\square}{\square}$. The fractional part of the mixed number must be less than 1. (For example, $4\frac{2}{3}$). What is the difference between the largest and smallest possible mixed number that can be formed?
(**A**) $4\frac{3}{5}$ (**B**) $4\frac{9}{20}$ (**C**) $4\frac{3}{10}$ (**D**) $4\frac{4}{15}$ (**E**) $4\frac{7}{20}$

Solution
The largest possible number that Michael can form is $5\frac{3}{4}$ while the smallest is $1\frac{2}{5}$.
The difference is $5\frac{3}{4} - 1\frac{2}{5} = 4\frac{7}{20}$. ANSWER: (E)

16. Suppose that x^* means $\frac{1}{x}$, the reciprocal of x. For example, $5^* = \frac{1}{5}$. How many of the following statements are true?

(i) $2^* + 4^* = 6^*$
(ii) $3^* \times 5^* = 15^*$
(iii) $7^* - 3^* = 4^*$
(iv) $12^* \div 3^* = 4^*$

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

Solution
(i) $\frac{1}{2} + \frac{1}{4} = \frac{3}{4} \neq \frac{1}{6}$, (i) is not true

(ii) $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$, (ii) is true

(iii) $\frac{1}{7} - \frac{1}{3} = \frac{3}{21} - \frac{7}{21} = \frac{4}{21} \neq \frac{1}{4}$, (iii) is not true

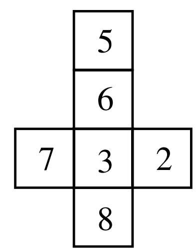
(iv) $\frac{\frac{1}{12}}{\frac{1}{3}} = \frac{1}{12} \times \frac{3}{1} = \frac{1}{4}$, (iv) is true

Only two of these statements is correct. ANSWER: (C)

17. In a ring toss game at a carnival, three rings are tossed over any of three pegs. A ring over peg *A* is worth *one* point, over peg *B three* points and over peg *C five* points. If all three rings land on pegs, how many different point totals are possible? (It is possible to have more than one ring on a peg.)
(**A**) 12 (**B**) 7 (**C**) 10 (**D**) 13 (**E**) 6

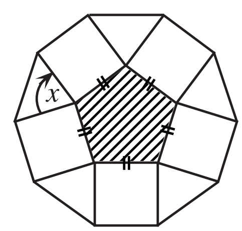
Solution
The lowest possible score is 3 and the highest is 15.
It is not possible to get an even score because this would require 3 odd numbers to add to an even number. Starting at a score of 3, it is possible to achieve every odd score between 3 and 15. This implies that 3, 5, 7, 9, 11, 13, and 15 are possible scores.
There are 7 possible scores. ANSWER: (B)

18. The figure shown is folded to form a cube. Three faces meet at each corner. If the numbers on the three faces at a corner are multiplied, what is the largest possible product?
(**A**) 144 (**B**) 168 (**C**) 240
(**D**) 280 (**E**) 336



Solution
When the figure is folded to make a cube, the numbers 8 and 6 are on opposite faces so that it is NOT possible to achieve $8 \times 7 \times 6$ or 336. It is possible, however, to have the sides with 5, 7 and 8 meet at a corner which gives the answer $5 \times 7 \times 8$ or 280. ANSWER: (D)

19. A regular pentagon has all sides and angles equal. If the shaded pentagon is enclosed by squares and triangles, as shown, what is the size of angle *x*?
(**A**) 75° (**B**) 108° (**C**) 90°
(**D**) 60° (**E**) 72°



Solution
Since a pentagon can be divided into three triangles, the sum of the angles in the pentagon is $3 \times 180° = 540°$. Since the angles in a regular pentagon are all equal, each one is $540° \div 5 = 108°$. The sum of the angles of each vertex of the pentagon is $x + 90° + 90° + 108° = 360°$.
Therefore $x = 72°$. ANSWER: (E)

20. Three playing cards are placed in a row. The club is to the right of the heart and the diamond. The 5 is to the left of the heart. The 8 is to the right of the 4. From left to right, the cards are
(**A**) 4 of hearts, 5 of diamonds, 8 of clubs
(**B**) 5 of diamonds, 4 of hearts, 8 of clubs
(**C**) 8 of clubs, 4 of hearts, 5 of diamonds
(**D**) 4 of diamonds, 5 of clubs, 8 of hearts
(**E**) 5 of hearts, 4 of diamonds, 8 of clubs

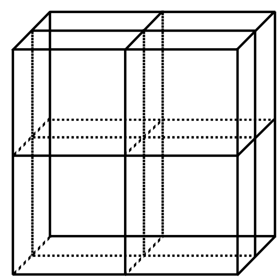
Solution
Since the club is to the right of the heart and diamond we know that the order is either heart, diamond, club or diamond, heart, club.
It is given that the 5 is to the left of the heart so this card must be the 5 of diamonds.
The order is 5 of diamonds, heart, club. Since the 8 is to the right of the 4, the heart must be a 4 and the club an 8.
The correct choice is B. ANSWER: (B)

Part C

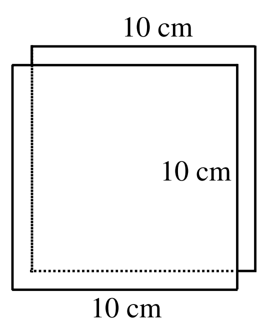
21. The number 315 can be written as the product of two odd integers each greater than 1. In how many ways can this be done?
(**A**) 0 (**B**) 1 (**C**) 3 (**D**) 4 (**E**) 5

Solution
Factoring 315 into primes, we find that $315 = 3 \times 3 \times 5 \times 7$.
The factorization of 315 as the product of 2 odd integers is 3×105, 5×63, 7×45, 9×35, and 15×21.
There are 5 possible factorizations. ANSWER: (E)

22. A cube measures $10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm}$. Three cuts are made parallel to the faces of the cube as shown creating eight separate solids which are then separated. What is the increase in the total surface area?
(**A**) 300 cm^2 (**B**) 800 cm^2 (**C**) 1200 cm^2
(**D**) 600 cm^2 (**E**) 0 cm^2



Solution
One cut increases the surface area by the equivalent of two $10 \text{ cm} \times 10 \text{ cm}$ squares or 200 cm^2.
Then the three cuts produce an increase in area of $3 \times 200 \text{ cm}^2$ or 600 cm^2.



ANSWER: (D)

23. If the sides of a triangle have lengths 30, 40 and 50, what is the length of the shortest altitude?
(**A**) 20 (**B**) 24 (**C**) 25 (**D**) 30 (**E**) 40

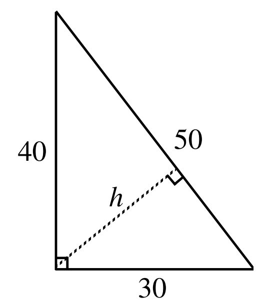
Solution

Since $30^2+40^2=50^2$ this is a right-angled triangle with a hypotenuse of 50 units.

The area of the triangle is $\frac{30\times 40}{2}$ or 600 sq. units.

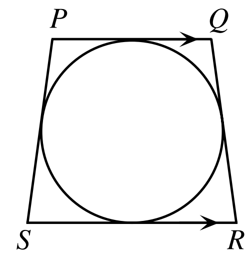
If we draw a perpendicular from the right angle and call this height, h, an expression for the area is $\frac{1}{2}(h)(50)=25h$.

Equating the two, we have $25h=600$ or $h=24$ which is the length of the shortest altitude.



ANSWER: (B)

24. A circle is inscribed in trapezoid $PQRS$.
If $PS=QR=25$ cm, $PQ=18$ cm and $SR=32$ cm, what is the length of the diameter of the circle?
(**A**) 14 (**B**) 25 (**C**) 24
(**D**) $\sqrt{544}$ (**E**) $\sqrt{674}$



Solution

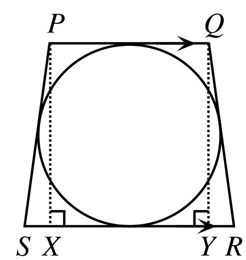
We start by drawing perpendiculars from P and Q to meet SR at X and Y respectively.

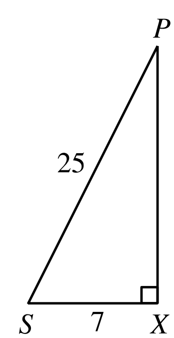
By symmetry, we see that $XY=PQ=18$. We also note that $SX=YR$ which means that $SX=YR=\frac{32-18}{2}=7$.

By applying the Pythagorean Theorem in ΔPXS we find,

$$(PX)^2+7^2=25^2$$
$$(PX)^2=576$$
$$PX=24$$

The diameter of the circle is thus 24 cm.





ANSWER: (C)

25. A sum of money is to be divided among Allan, Bill and Carol. Allan receives \$1 plus one-third of what is left. Bill then receives \$6 plus one-third of what remains. Carol receives the rest, which amounts to \$40. How much did Bill receive?
(**A**) \$26 (**B**) \$28 (**C**) \$30 (**D**) \$32 (**E**) \$34

Solution

After Allan had received his share, Bill received \$6 plus one-third of the remainder.
Since Carol gets the rest, she received two-thirds of the remainder, which is \$40.
Thus, one-third of the remainder is \$20.
The amount Bill receives is \$26. ANSWER: (A)