Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 19, 1997

Part A: Each question is worth 5 credits.

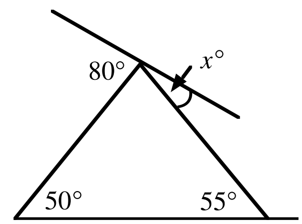
1. The value of $2\frac{1}{10}+3\frac{11}{100}$ is
   (**A**) 5.11 (**B**) 5.111 (**C**) 5.12 (**D**) 5.21 (**E**) 5.3

2. The value of $(1)^{10}+(-1)^{8}+(-1)^{7}+(1)^{5}$ is
   (**A**) 0 (**B**) 1 (**C**) 2 (**D**) 16 (**E**) 4

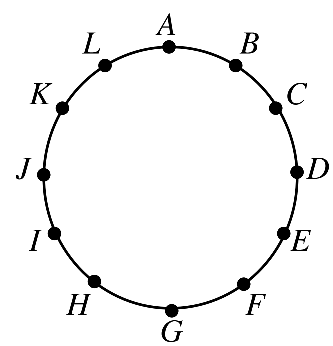
3. An integer is multiplied by 2 and the result is then multiplied by 5. The final result could be
   (**A**) 64 (**B**) 32 (**C**) 12 (**D**) 25 (**E**) 30

4. The greatest number of Mondays that can occur in 45 consecutive days is
   (**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

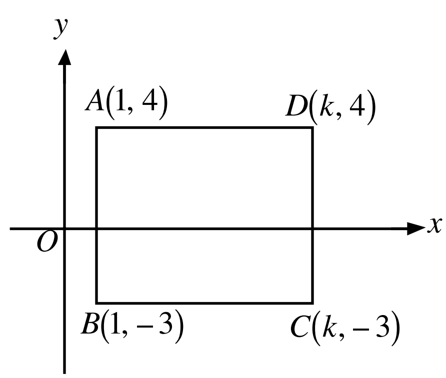
5. The value of x is
   (**A**) 25 (**B**) 30 (**C**) 50
   (**D**) 55 (**E**) 20



6. Twelve balloons are arranged in a circle as shown. Counting clockwise, every third balloon is popped, with C the first one popped. This process continues around the circle until two unpopped balloons remain. The last two remaining balloons are
   (**A**) B, H (**B**) B, G (**C**) A, E
   (**D**) E, J (**E**) F, K



7. In the diagram, rectangle $ABCD$ has area 70 and k is positive. The value of k is
   (**A**) 8 (**B**) 9 (**C**) 10
   (**D**) 11 (**E**) 12



8. If p, q, r, s , and t are numbers such that $r < s$, $t > q$, $q > p$, and $t < r$, which of these numbers is greatest?
(**A**) t (**B**) s (**C**) r (**D**) q (**E**) p

9. The sum of seven consecutive integers is 77. The smallest of these integers is
(**A**) 5 (**B**) 7 (**C**) 8 (**D**) 11 (**E**) 14

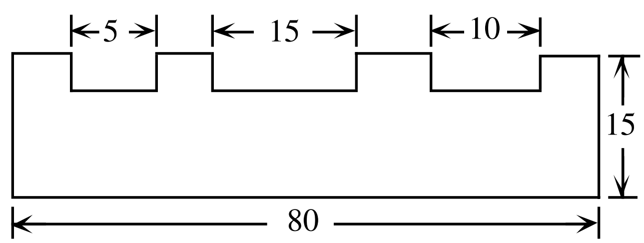
10. Each of the numbers 1, 2, 3, and 4 is assigned, in some order, to p, q, r, and s. The largest possible value of $p^q + r^s$ is
(**A**) 12 (**B**) 19 (**C**) 66 (**D**) 82 (**E**) 83

Part B: Each question is worth 6 credits.

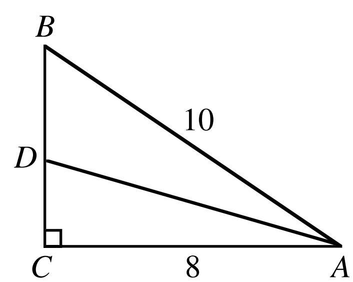
11. In the chart, the products of the numbers represented by the letters in each of the rows and columns are given. For example, $xy = 6$ and $xz = 12$. If x, y, z, and w are integers, what is the value of xw?
(**A**) 150 (**B**) 300 (**C**) 31
(**D**) 75 (**E**) 30

x	y	6
z	w	50
12	25	

12. Three small rectangles, of the same depth, are cut from a rectangular sheet of metal. The area of the remaining piece is 990. What is the depth of each cut?
(**A**) 8 (**B**) 7 (**C**) 6
(**D**) 5 (**E**) 4



13. Triangle ABC is right-angled with $AB = 10$ and $AC = 8$. If $BC = 3DC$, then AD equals
(**A**) 9 (**B**) $\sqrt{65}$ (**C**) $\sqrt{80}$
(**D**) $\sqrt{73}$ (**E**) $\sqrt{68}$



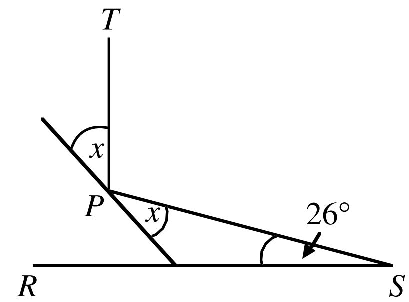
14. The digits 1, 2, 3, 4 can be arranged to form twenty-four different four digit numbers. If these twenty-four numbers are then listed from smallest to largest, in what position is 3142?
(**A**) 13th (**B**) 14th (**C**) 15th (**D**) 16th (**E**) 17th

15. The product of 20^{50} and 50^{20} is written as an integer in expanded form. The number of zeros at the end of the resulting integer is

(**A**) 70 (**B**) 71 (**C**) 90 (**D**) 140 (**E**) 210

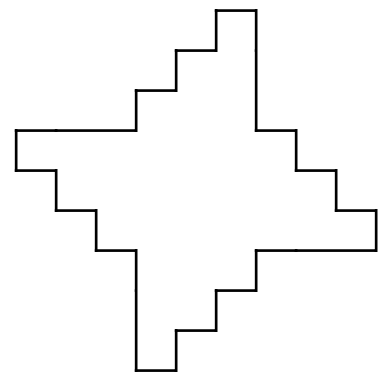
16. A beam of light shines from point S, reflects off a reflector at point P, and reaches point T so that PT is perpendicular to RS. Then x is

(**A**) 32° (**B**) 37° (**C**) 45°
(**D**) 26° (**E**) 38°



17. In the diagram adjacent edges are at right angles. The four longer edges are equal in length, and all of the shorter edges are also equal in length. The area of the shape is 528. What is the perimeter?

(**A**) 132 (**B**) 264 (**C**) 92
(**D**) 72 (**E**) 144



18. If $\frac{30}{7} = x + \frac{1}{y + \frac{1}{z}}$, where x, y, and z are positive integers, then what is the value of $x + y + z$?

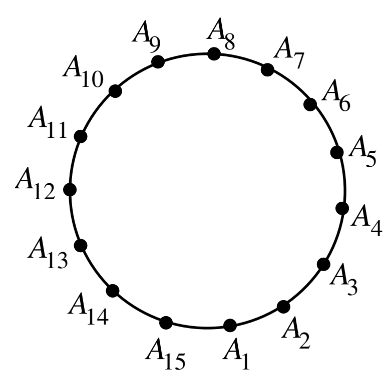
(**A**) 13 (**B**) 9 (**C**) 11 (**D**) 37 (**E**) 30

19. If $x^2yz^3 = 7^4$ and $xy^2 = 7^5$, then xyz equals

(**A**) 7 (**B**) 7^2 (**C**) 7^3 (**D**) 7^8 (**E**) 7^9

20. On a circle, fifteen points $A_1, A_2, A_3, \ldots, A_{15}$ are equally spaced. What is the size of angle $A_1A_3A_7$?

(**A**) 96° (**B**) 100° (**C**) 104°
(**D**) 108° (**E**) 120°



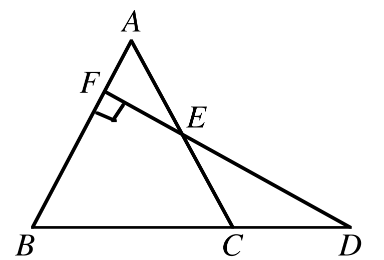
Part C: Each question is worth 8 credits.

21. If $\dfrac{\left(\frac{a}{c}+\frac{a}{b}+1\right)}{\left(\frac{b}{a}+\frac{b}{c}+1\right)}=11$, where a, b, and c are positive integers, the number of different ordered triples (a, b, c) such that $a+2b+c\leq 40$ is

(**A**) 33 (**B**) 37 (**C**) 40 (**D**) 42 (**E**) 45

22. In the diagram, ΔABC is equilateral, $BC=2CD$, $AF=6$, and DEF is perpendicular to AB. What is the area of quadrilateral $FBCE$?

(**A**) $144\sqrt{3}$ (**B**) $138\sqrt{3}$ (**C**) $126\sqrt{3}$
(**D**) $108\sqrt{3}$ (**E**) $66\sqrt{3}$



23. Given the set $\{1, 2, 3, 5, 8, 13, 21, 34, 55\}$, how many integers between 3 and 89 cannot be written as the sum of exactly two elements of the set?

(**A**) 51 (**B**) 57 (**C**) 55 (**D**) 34 (**E**) 43

24. In a convex polygon, exactly five of the interior angles are obtuse. The largest possible number of sides for this polygon is

(**A**) 7 (**B**) 8 (**C**) 9 (**D**) 10 (**E**) 11

25. In triangle ABC, $BR=RC$, $CS=3SA$, and $\frac{AT}{TB}=\frac{p}{q}$. If the area of ΔRST is twice the area of ΔTBR, then $\frac{p}{q}$ is equal to

(**A**) $\frac{2}{1}$ (**B**) $\frac{8}{3}$ (**C**) $\frac{5}{2}$
(**D**) $\frac{7}{4}$ (**E**) $\frac{7}{3}$