Centre for Education
in Mathematics and Computing

Euclid eWorkshop # 4

Solutions

SOLUTIONS

1. (a) Here $\sin 2\theta = -\dfrac{1}{2}$. Thus $2\theta = 210°$ and $\theta = 105°$.

   (b)

$$\begin{aligned}\cos^2(\theta) &= 1 - \sin^2(\theta)\\ 2(2\sin^2(\theta) - 1) &= 8\sin\theta - 5\\ 4\sin^2(\theta) - 8\sin\theta + 3 &= 0\\ (2\sin\theta - 1)(2\sin\theta - 3) &= 0\\ \sin\theta &= \frac{1}{2}, \frac{3}{2} \text{ but} |\sin\theta| \le 1\\ \text{So } \theta &= \frac{\pi}{6}, \frac{5\pi}{6}\end{aligned}$$

2. Let $\theta = \angle AMC$. Using the cosine law in $\triangle ABM$ gives

$$\begin{aligned}49 &= 9 + 25 - 30\cos(180° - \theta)\\ 15 &= -30\cos(180° - \theta)\\ \cos(180° - \theta) &= -\frac{1}{2}\\ \cos(\theta) &= -\cos(180° - \theta)\\ &= \frac{1}{2}\end{aligned}$$

   Using the cosine law in $\triangle AMC$ gives

$$\begin{aligned}AC^2 &= 9 + 36 - 36\cos(\theta)\\ &= 27\\ AC &= 3\sqrt{3}.\end{aligned}$$

3. Use the sine law: $\dfrac{BN}{\sin(108°)} = \dfrac{100}{\sin(25°)}$. But $\dfrac{MN}{BN} = \tan 32°$. So $MN = 100 \cdot \dfrac{\sin 108°}{\sin 25°} \cdot \tan 32° \approx 141$ m.

4. Since the area of the rectangle is $\dfrac{5\pi}{3}$, its height is 5. Since the cosine graph is symmetrical about the y-axis, $PO = OQ = \dfrac{\pi}{6}$. But $\cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$. So $k = \dfrac{10\sqrt{3}}{3}$.

5. Since the minimum point has a y coordinate of -2, the amplitude is $a = 2$. Also since the minimum occurs at $x = \dfrac{3\pi}{4}$ (rather than $\dfrac{3\pi}{2}$ where it is found for $\sin(x)$), $k = 2$. Therefore $\sin(2x) = \dfrac{1}{2}$ and $x = \dfrac{\pi}{12}$. Thus $D = (\dfrac{\pi}{12}, 1)$.

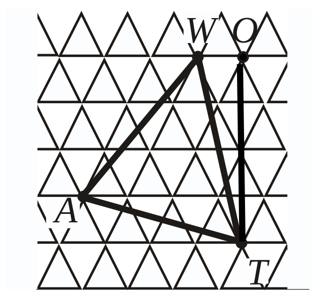
6. We let the side of the triangles opposite θ be a and side leg adjacent to θ be b. Then $\tan\theta = \dfrac{a}{b}$, $a - b = 3$ and $4\left(\dfrac{1}{2}ab\right) = 89 - 9 = 80$ and $b = \frac{40}{a}$. Thus $a - \dfrac{40}{a} = 3$ or $a^2 - 3a - 40 = 0$ which gives $a = 8$ or -5. Now

since a is positive, $a = 8$ and $b = 5$ and $\tan\theta = \dfrac{8}{5}$.

7. Using Pythagorean theorem, we find that $FA = 2$, $AC = \sqrt{2}$ and $FC = 2$. The cosine law in $\triangle FAC$ gives

$$\begin{aligned} FC^2 &= FA^2 + AC^2 - 2 \cdot FA \cdot AC \cdot \cos(\angle FAC) \\ 4 &= 4 + 2 - 2 \cdot 2 \cdot \sqrt{2}\cos(\angle FAC) \\ \cos(\angle FAC) &= \frac{1}{2\sqrt{2}}. \end{aligned}$$

8. Using item 4 from the toolkit, the height of each small equilateral triangle is $\frac{\sqrt{3}}{2}$. Let O be the vertex immediately to the right of W, and consider right-angled triangle WOT. Now OT is the height of four of the small triangles, thus $OT = 2\sqrt{3}$. Also $WO = 1$. By the Pythagorean theorem, we have $WT = \sqrt{1 + 4 \cdot 3} = \sqrt{13}$. Again using item 4 from the toolkit, we have the area of $\triangle WAT = \dfrac{13\sqrt{3}}{4}$.



9. The cosine law states

$$\begin{aligned} a^2 &= 64 + b^2 - 16b(\cos 60^\circ) \\ &= b^2 - 8b + 64 \\ &= (b-4)^2 + 48 \\ a^2 - (b-4)^2 &= 48 \\ (a+b-4)(a-b+4) &= 48 \end{aligned}$$

But $48 = 24 \cdot 2 = 12 \cdot 4 = 8 \cdot 6$, where we have only considered the even-even factorings of 48 since the 2 brackets on the left side must have the same parity (evenness or oddness). Thus $(a{,}b) = (13{,}15)$, (8,8), (7,5) or (7,3).