Centre for Education
in Mathematics and Computing

Euclid eWorkshop # 1

Solutions

SOLUTIONS

1. We have $\log_x(2 \cdot 4 \cdot 8) = 1 \Rightarrow \log_x(64) = 1 \Rightarrow x = 64$.

2. Since $12 = 2^2 \cdot 3$ it follows that

$$\begin{aligned} 2^{2(2x+1)}3^{2x+1} &= 2^{3x+7}3^{3x-4} \\ 2^{2(2x+1)-3x-7} &= 3^{3x-4-2x-1} \\ 2^{x-5} &= 3^{x-5} \end{aligned}$$

   Since the graphs of $y = 2^{x-5}$ and $y = 3^{x-5}$ intersect only at $x = 5$ and $y = 1$ it follows that $x = 5$ is the only solution.

3. This expression equals $\log_{10}\left(\frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdot \cdots \cdot \frac{200}{199}\right) = \log_{10}\frac{200}{2} = \log_{10} 100 = 2$.

4. The second equation states $xy^{-2} = 3^{-3}$ or $x^3y^{-6} = 3^{-9}$. Dividing the first equation by this new equation we can eliminate x:

$$\begin{aligned} \frac{x^3y^5}{x^3y^{-6}} &= \frac{2^{11}3^{13}}{3^{-9}} \\ y^{11} &= 2^{11} \cdot 3^{22} \\ y &= 18. \end{aligned}$$

   Then $x = \frac{y^2}{27} = 12$.

5. $\log_8(18) = \log_8 2 + \log_8 9 = \frac{1}{3} + 2k$.

6. We express the logarithms in exponential form to arrive at: $2x = 2^y$ and $x = 4^y$. Thus

$$\begin{aligned} 2^y &= 2(4^y) \\ 2^y &= 2^{2y+1} \\ 1 &= 2^{y+1} \end{aligned}$$

   Thus $y = -1$ and $x = \frac{1}{4}$.

7. We note first that $x = a^y$ for all points on the curve. The midpoint of AB is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

   Since we draw a horizontal line from the midpoint, $y_3 = \frac{y_1 + y_2}{2}$. So

$$\begin{aligned} (x_3)^2 &= (a^{y_3})^2 \\ &= (a^{\frac{y_1+y_2}{2}})^2 \\ &= (a^{y_1})(a^{y_2}) \\ &= x_1x_2. \end{aligned}$$

8. We have $1 = a(2^r)$ and $4 = a(32^r)$. Dividing the second equation by the first gives $4 = (16^r)$ and $r = \frac{1}{2}$.

9. Factoring the equation $2^{x+3} + 2^x = 3^{y+2} - 3^y$ gives

$$\begin{aligned}(2^3+1)2^x &= (3^2-1)3^y \\ 9 \cdot 2^x &= 8 \cdot 3^y \\ 3^2 \cdot 2^x &= 2^3 \cdot 3^y \\ 2^{x-3} &= 3^{y-2}\end{aligned}$$

Since x and y are integers, we have$x = 3$ and $y = 2$.

10. If $f(x) = 2^{4x-2}$ then $f(x) \cdot f(1-x) = 2^{4x-2} \cdot 2^{4(1-x)-2} = 2^{4x-2+4-4x-2} = 2^0 = 1$.

11. Observe that the argument of both logs must be positive, so $x > 6$. Now

$$\begin{aligned}\log_5(x-2) + \log_5(x-6) &= 2 \\ \log_5((x-2)(x-6)) &= 2 \\ (x-2)(x-6) &= 25 \\ x &= 4 \pm \sqrt{29}\end{aligned}$$

However since $x > 6$, $x = 4 + \sqrt{29}$.

12. If a, b, c are in geometric sequence, then $\frac{a}{b} = \frac{b}{c}$. From this result it follows that $\log_x \left(\frac{a}{b}\right) = \log_x \left(\frac{b}{c}\right)$ which implies $\log_x(a) - \log_x(b) = \log_x(b) - \log_x(c)$; therefore the required logarithms are in arithmetic sequence.

If $\log_x a, \log_x b, \log_x c$ form an arithmetic sequence, then

$$\begin{aligned}\log_x a - \log_x b &= \log_x b - \log_x c \\ \log_x(\frac{a}{b}) &= \log_x(\frac{b}{c}) \\ \frac{a}{b} &= \frac{b}{c} \quad \text{since the log function takes on each value only once}\end{aligned}$$

Thus a, b, c are in geometric sequence.