The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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2021 Team Up Challenge

June 2021

Solutions

Team Paper

1. From the graph, we can see that Ana spent $2 + 1 + 0.5 + 1 + 0 + 0.5 + 2.5 = 7.5$ hours playing soccer.

ANSWER: 7.5

2. Since the four sides of a square are equal in length and the perimeter is 12 cm, then each side has length $\frac{12}{4} = 3$ cm.

ANSWER: 3 cm

3. If the symbol $\triangle$ represents the number 5, then the expression becomes $5 \times 5 + 5$ which is equal to 30.

ANSWER: 30

4. The character starts by moving 5 steps.
Inside the loop, the character moves 3 steps. Since the character repeats the loop 8 times, this is a total of $8 \times 3 = 24$ steps.
In total, the character has moved $5 + 24 = 29$ steps.

ANSWER: 29

5. Since $125 \times 40 = 5000$ (or $5000 \div 40 = 125$), then Jeff would need to type for 125 minutes.

ANSWER: 125

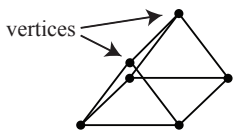
6. Exactly 3 of the 8 equal sections are labelled with the letter B. Therefore, the probability that the spinner lands on B is $\frac{3}{8}$ or 37.5%.

ANSWER: $\frac{3}{8}$ or 37.5%

7. Recall that a prime number is an integer greater than 1 whose only divisors are 1 and itself.
The prime numbers between 6 and 30 are: 7, 11, 13, 17, 19, 23, and 29.
Thus, there are 7 integers between 6 and 30 which are prime numbers.

ANSWER: 7

8. Every triangular prism has exactly 6 vertices, as shown in the diagram.

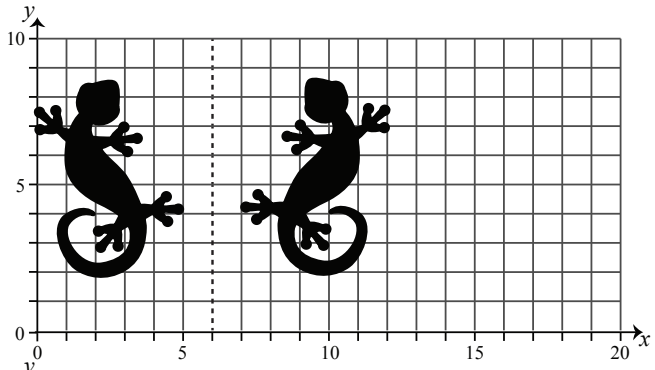


ANSWER: 6

9. The pattern repeats after every 6 shapes, so we want to find the largest multiple of 6 that is less than 55. This is 54. The pattern will start again after the 54^{th} shape. The 55^{th} shape will then be the first shape in the pattern, which is ◯ (the circle).

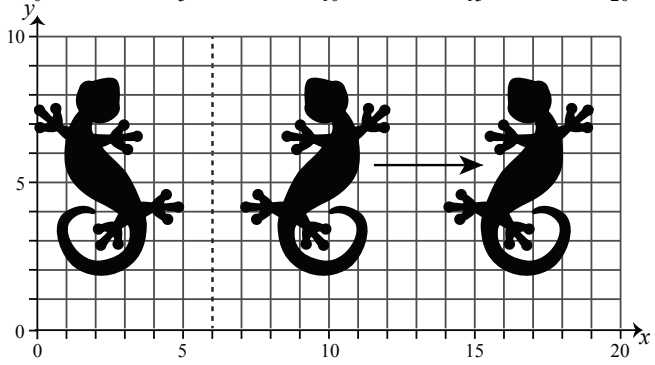
ANSWER: ◯

10. We start by reflecting the lizard over the dotted line as shown.



We then translate the lizard 7 units to the right, as shown.

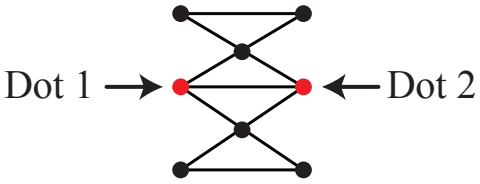
The tail now has coordinates $(17, 4)$.



ANSWER: $(17, 4)$

11. It is possible to trace over all the lines exactly once in Picture B, Picture C and Picture E.

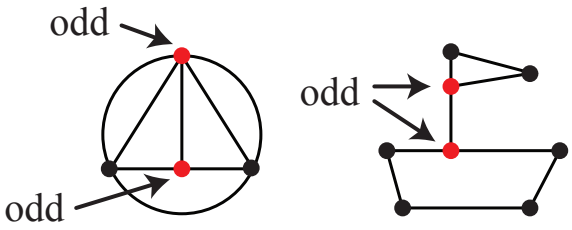
First we will look at Picture B.



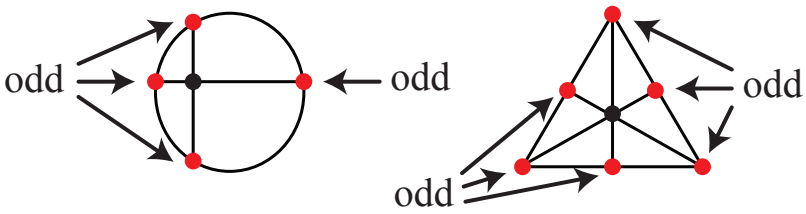
One way to do trace all the lines as described is to start from Dot 1 and trace over the line connecting Dot 1 to Dot 2 and then trace over all the remaining lines, finishing at Dot 2.

There are other ways to do this, but the important thing is that we have to start at either Dot 1 or Dot 2 since these are the only dots with an odd number of lines coming out of them. Every time we pass through a dot (other than at the beginning or end), we use up an even number of lines. So, the only way to use up all the lines for a dot with an odd number of lines coming out of it is to make sure that dot is the first or last dot.

Notice that Picture C and Picture E each have exactly two dots with an odd number of edges coming out of them. We can trace over all the lines exactly once in each of these pictures by starting at one of these dots and finishing at the other.



Picture A and Picture D each have more than two dots with an odd number of lines coming out them, so there's no way to trace over all the lines in these pictures exactly once.



Therefore, it is possible to trace over every line, as described, in Pictures B, C, and E.

ANSWER: B, C, E

12. Since the known numbers in the list are all different, then if the unknown number is different than each of these, there is no single mode. Therefore, the unknown number must be equal to one of the known numbers. It follows that the unknown number will also be the mode.
If we arrange the known numbers in increasing order, we have: 2, 3, 5, 6, 9.
If the unknown number is 2 or 3, then the median of the six numbers would be $\frac{3+5}{2} = 4$ which is not possible, since the median does not equal the mode.
If the unknown number is 6 or 9, then the median of the six numbers would be $\frac{5+6}{2} = 5.5$ which is not possible, since the median does not equal the mode.
If the unknown number is 5, then the median of the six numbers is 5 and the mode of the six numbers is also 5.
Therefore, the unknown number is 5.

ANSWER: 5

13. We begin by considering the positive integers between 1 and 100. From 1 to 69, the digit 7 is written 7 times. From 70 to 79, the digit 7 is written 11 times. From 80 to 100, the digit 7 is written 2 times. Thus, from 1 to 100, the digit 7 is written 20 times.

The next 10 times Jonah writes the digit 7 are as follows:

$$107, 117, 127, 137, 147, 157, 167, 170, 171, 172$$

Therefore, Jonah's favourite number is 172.

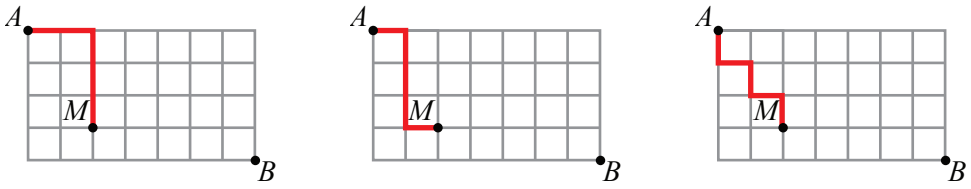
ANSWER: 172

14. Bea, Diane, and Gene have each met 7 people, therefore Hans has met Bea, Diane and Gene. Edgar and Foster have both met 3 people. Since they have met Bea, Diane, and Gene, then they have not met anyone else. So, Hans has not met Edgar or Foster.
Amad has met 5 people. Since Amad cannot have met Edgar or Foster, then Amad has met Cho and Hans.
Cho has met 4 people: Amad, Bea, Diane, and Gene.
Therefore, Hans has met 4 people: Amad, Bea, Diane and Gene.

ANSWER: 4

15. Since the robot must travel though M we can count the number of paths from A to M and then count the number of paths from M to B.

We begin by considering the following three paths from A to M.



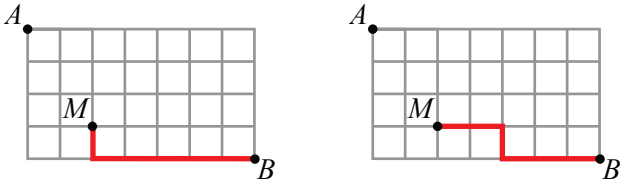
Notice that these paths involve two moves right and three moves down. In fact, every path from A to M will involve exactly two moves right and three moves down. What makes each path different is the order in which the robot makes these moves. For example, if R means move right and D means move down, then the sequence of moves R, R, D, D, D is different from the sequence of moves R, D, R, D, D.
Counting the number of different possible sequences of moves, we get:

R, R, D, D, D R, D, R, D, D R, D, D, R, D R, D, D, D, R D, R, R, D, D

D, R, D, R, D D, R, D, D, R D, D, R, R, D D, D, R, D, R D, D, D, R, R

There are a total of 10 ways the robot can get from A to M.

Similarly, we can consider the following two paths from M to B.



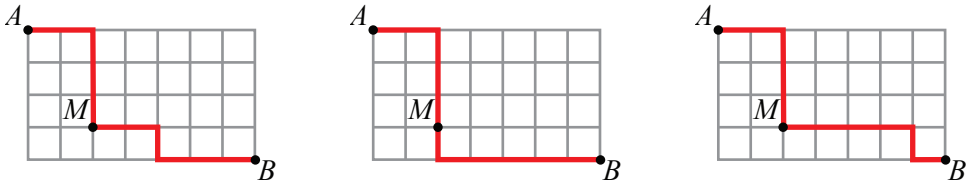
Notice that these paths involve five moves right and one move down. In fact, every path from M to B will involve exactly five moves right and one move down. What makes each path different is the order in which the robot makes these moves. Counting the number of different orders that are possible, we get:

R, R, R, R, R, D R, R, R, R, D, R R, R, R, D, R, R

R, R, D, R, R, R R, D, R, R, R, R D, R, R, R, R, R

There are a total of 6 ways the robot can get from M to B.

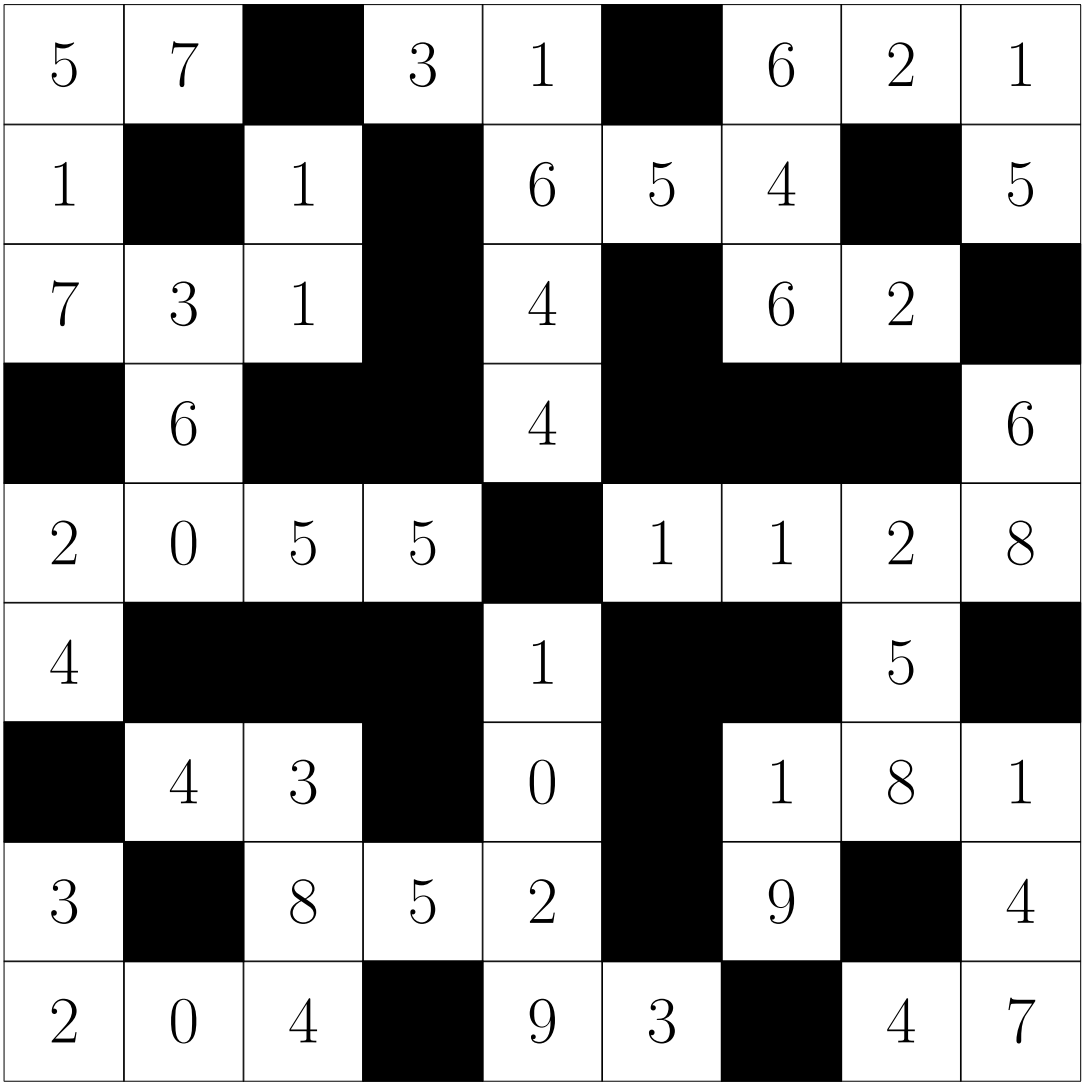
We now consider three of the possible paths from A to B through M.



Notice that, in each of these three paths, the robot takes the same path from A to M, but then takes a different path from M to B. What makes each path different is how the robot moves after he reaches M. Since each of the 10 paths from A to M can be paired with each of the six paths from M to B, there are a total of $10 \times 6 = 60$ paths the robot can take from A to B.

ANSWER: 60

Crossnumber Puzzle



Across

1. The range is $\boxed{204} - \boxed{147} = 57$.

2. 50% of $\boxed{62} = 0.5 \times \boxed{62} = 31$.

4. Since $\dfrac{23}{\boxed{24}} = \dfrac{621}{648}$, the number is 621.

7. From the grid, the hundreds digit of this number is 6. The only multiple of 109 with a hundreds digit of 6 is 654.

8. The sum is $110 + \boxed{621} = 731$.

10. July and August each have 31 days. Thus the total number of days is 62.

12. The result is $45 \times 45 + 30 = 2055$.

13. The product is $12 \times 94 = 1128$.

16. From the grid, the ones digit of this number is 3. Since the only factors of $\boxed{731}$ are 1, 17, 43, and 731, the only two-digit factor with a ones digit of 3 is 43.

18. From the grid, the ones digit of this number is 1. It follows that the hundreds digit is also 1. The tens digit will be determined from the clue to $\boxed{14 \text{ DOWN}}$.

21. From the grid, the hundreds digit is 8 and the ones digit is 2. The median of these two numbers is 5. Thus, the number is 852

22. A dozen is a set of 12. So seventeen dozen is $17 \times 12 = 204$.

23. The sum of the digits in this number must be 12, since the digit sum of $\boxed{1128}$ is 12. From the grid, the tens digit of this number is 9. Thus, the number is 93.

24. The smallest prime number greater than $\boxed{43}$ is 47.

Down

1. The number is $16 \times \boxed{32} + 5 = 517$.

3. The number is $\frac{4}{5}$ of $\boxed{2055} = 0.8 \times \boxed{2055} = 1644$.

4. The digits are determined from the clues to $\boxed{4 \text{ ACROSS}}$, $\boxed{7 \text{ ACROSS}}$, and $\boxed{10 \text{ ACROSS}}$.

5. An odd multiple of 5 has a ones digit of 5. The tens digit will be determined from the clue to $\boxed{4 \text{ ACROSS}}$.

6. A rectangle with area $\boxed{517}$ and width $\boxed{47}$ has length $\boxed{517} \div \boxed{47} = 11$.

9. In a square, each angle is equal to 90°. The sum of four angles, in degrees, is $90 \times 4 = 360$.

11. The number is $\boxed{62} + 6 = 68$.

12. From the grid, the tens digit of this number is 2. The only multiple of 8 with a tens digit of 2 is 24.

14. The digits of $\boxed{852}$ in reverse order are 258.

15. Since $1000 \div 7 = 142.8$, the largest 3-digit multiple of 7 is $142 \times 7 = 994$. Then $994 + 35 = 1029$.

17. The mean is $\dfrac{\boxed{621} + \boxed{147}}{2} = 384$.

18. The number is $\boxed{43} - \boxed{24} = 19$.

19. There are 7 days in a week, so $\boxed{1029}$ days is $\boxed{1029} \div 7 = 147$ weeks.

20. The value is $14 + 0.5 \times 36 = 14 + 18 = 32$.

Logic Puzzle
Start by considering the second, eighth and first clues (in that order).

(2) Dorothy and Sophia are the only people to ever stand in Position 2
Although we do not yet know which person occupies which Position 2 in each photo, we can make note that one of them does.

(8) The order for the garden photo is the reverse of the order for the driveway photo
Since either Dorothy or Sophia occupy the second position in these photos, it follows that they must occupy the third position as well. As a result, Blanche and Rose must somehow occupy Positions 1 and 4 in these photos.

(1) Blanche is standing in Position 1 in four of the five photos
Since the garden photo is the reverse of the driveway photo it is not possible for Blanche to be in Position 1 in both of these photos. Thus, in order to be standing in Position 1 in four photos, she must be standing in Position 1 in the park, kitchen and basement photos. Then, the fourth photo with Blanche in Position 1 will be either the garden or driveway photo.

The following partially-completed table contains what we have determined from these three clues.

		Position in Photo			
		1	2	3	4
Location	Park	**Blanche**	Dorothy/Sophia		
	Kitchen	**Blanche**	Dorothy/Sophia		
	Garden	Blanche/Rose	Dorothy/Sophia	Dorothy/Sophia	Blanche/Rose
	Driveway	Blanche/Rose	Dorothy/Sophia	Dorothy/Sophia	Blanche/Rose
	Basement	**Blanche**	Dorothy/Sophia		

Next, we consider the third clue.

(3) Rose is standing in the same position for the photos in the basement and the driveway, but she isn't standing in that position in any other photos.
We previously determined that Rose is standing in either Position 1 or Position 4 for the driveway photo. Since she must be standing in the same position for the basement photo it follows that she must be standing in Position 1 or Position 4 in the basement photo. But, Blanche is standing in Position 1 in the basement photo, and so Rose must be standing in Position 4 for both the basement and the driveway photos.

Now that we know Rose is standing in Position 4 in the driveway photo, we also know that Blanche is in Position 1. Furthermore, since the garden photo is the reverse of the driveway photo, we know that in the garden photo Rose is in Position 1 and Blanche is in Position 4.

Furthermore, since Rose is not standing in the same position in the park and kitchen photos as she is in the driveway/basement photos, it follows that Rose is not standing in Position 4 in either the park or the kitchen photos. We already know that Blanche is in Position 1 in both photos, and that either Dorothy or Sophia are in Position 2 in both photos. Therefore, Rose must be standing in Position 3 for both the park and the kitchen photos.

		Position in Photo			
		1	2	3	4
Location	Park	**Blanche**	Dorothy/Sophia	**Rose**	
	Kitchen	**Blanche**	Dorothy/Sophia	**Rose**	
	Garden	**Rose**	Dorothy/Sophia	Dorothy/Sophia	**Blanche**
	Driveway	**Blanche**	Dorothy/Sophia	Dorothy/Sophia	**Rose**
	Basement	**Blanche**	Dorothy/Sophia		**Rose**

Now, we consider the fourth clue.

(4) Dorothy is never standing in Position 1 or Position 4
In both the park and the kitchen photo, the only possible positions left for Dorothy to stand in are Positions 2 and 4. Since Dorothy can never stand in Position 4, she must be standing in Position 2 for both of these photos. This leaves Sophia standing in Position 4 for both the park and the kitchen photos.

		Position in Photo			
		1	2	3	4
Location	Park	**Blanche**	**Dorothy**	**Rose**	**Sophia**
	Kitchen	**Blanche**	**Dorothy**	**Rose**	**Sophia**
	Garden	**Rose**	Dorothy/Sophia	Dorothy/Sophia	**Blanche**
	Driveway	**Blanche**	Dorothy/Sophia	Dorothy/Sophia	**Rose**
	Basement	**Blanche**	Dorothy/Sophia		**Rose**

Notice that we have now determined the order in which the siblings are standing for the park and kitchen photos. That means, that the sixth clue is not actually needed. But, we can confirm that it is in fact satisfied, since the siblings are standing in the same order for the park and kitchen photos.

Now, we can use the fifth clue.

(5) Sophia is standing in Position 2 exactly twice.
Since the garden photo is the reverse of the driveway photo, Sophia is in Position 2 exactly once in these two photos. As a result, to be in Position 2 exactly twice, she must be in Position 2 in the basement photo. As a result, Dorothy must be standing in Position 3 in the basement photo.

		Position in Photo			
		1	2	3	4
Location	Park	**Blanche**	**Dorothy**	**Rose**	**Sophia**
	Kitchen	**Blanche**	**Dorothy**	**Rose**	**Sophia**
	Garden	**Rose**	Dorothy/Sophia	Dorothy/Sophia	**Blanche**
	Driveway	**Blanche**	Dorothy/Sophia	Dorothy/Sophia	**Rose**
	Basement	**Blanche**	**Sophia**	**Dorothy**	**Rose**

Finally, the seventh clue tells us where Dorothy and Sophia are standing in the garden and driveway photos.

(7) In three of the four photos where Blanche is standing in Position 1, Dorothy is standing next to her.
This clue is equivalent to saying that in three of the four photos where Blanche is standing in Position 1, Dorothy is standing in Position 2. Currently, we have determined that Dorothy is standing in Position 2 in exactly two photos where Blanche is standing in Position 1. Therefore, Dorothy must be standing in Position 2 in the driveway photo (where Blanche is standing in Position 1). As a result, Sophia will be standing in Position 3.

Since the garden photo is the reverse of the driveway photo, we know that in the garden photo Sophia is in Position 2 and Dorothy is in Position 3. This completes the logic puzzle.

		Position in Photo			
		1	2	3	4
Location	Park	**Blanche**	**Dorothy**	**Rose**	**Sophia**
	Kitchen	**Blanche**	**Dorothy**	**Rose**	**Sophia**
	Garden	**Rose**	**Sophia**	**Dorothy**	**Blanche**
	Driveway	**Blanche**	**Dorothy**	**Sophia**	**Rose**
	Basement	**Blanche**	**Sophia**	**Dorothy**	**Rose**

Relay

(Note: Where possible, the solutions are written as if the value of N is not initially known, and then N is substituted at the end.)

Practice Relay

P1: Evaluating, $2 + 0 + 2 + 1 = 5$.

P2: The numbers 16, 450, and 34 are the only even numbers in the list. So we know that there are 3 even numbers if N is odd and 4 even numbers if N is even.
Since the answer to the previous question is 5, then N is odd, and so there are 3 even numbers in the list.

P3: The sum of the side lengths is $7 + 4 + 3 + N = 14 + N$.
Since the answer to the previous question is 3, then $N = 3$, and so $14 + N = 14 + 3 = 17$.

P4: Since 2 people get off the bus at the first stop and 9 people get on the bus at the second stop, there are $9 - 2 = 7$ more people on the bus after the second stop. In total, there are $N + 7$ people on the bus.
Since the answer to the previous question is 17, then $N = 17$, and so $17 + 7 = 24$.

ANSWER: $5, 3, 17, 24$

Relay A

P1: A vertical line can be drawn to separate the shape into two rectangles. One rectangle has an area of $12 \times 5 = 60$. The other rectangle has an area of $10 \times 5 = 50$. The total area is then $50 + 60 = 110$.

P2: If N newspapers represents two-thirds of the deliveries, then $\frac{N}{2}$ represents one-third of the deliveries. Then, $3 \times \left(\frac{N}{2}\right)$ represents the total number of papers that Ann delivered.
Since the answer to the previous question is 110, then $N = 110$, and so
$3 \times \left(\frac{N}{2}\right) = 3 \times \left(\frac{110}{2}\right) = 3 \times 55 = 165$.

P3: In increasing order, the known numbers are: 45, 62, 78, 92, 99, 101, 103, 123, 156, 196.
If N is less than or equal to 99, the middle number is 99.
If N is greater than 99 but less than 101, the middle number is N.
If N is greater than or equal to 101, the middle number is 101.
Since the answer from the previous question is 165, then $N = 165$, and so since $165 > 101$ the middle number is 101.

P4: If there were N hats at the end of the day, then before the third concert there were $2 \times N$ hats. Then, before the second concert there must have been $2 \times 2 \times N = 4 \times N$ hats and before the first concert there must have been $2 \times 4 \times N = 8 \times N$ hats.
Since the answer from the previous question is 101, then $N = 101$, and so the musician started with $8 \times 101 = 808$ hats.

ANSWER: $110, 165, 101, 808$

Relay B

P1: There are 7 children who are 8 years old and 2 children who are 9 years old. Thus, there are $7 + 2 = 9$ children older than 7.

P2: In total, Pierre has $(2 \times \$1) + (1 \times \$2) + (2 \times \$5) + (4 \times \$10) = \$54$.
When divided among N people, each person will get $\frac{\$54}{N}$.
Since the answer from the previous question is 9, then $N = 9$, and so $\frac{\$54}{N} = \frac{\$54}{9} = 6$.

P3: The terms in this sequence are: 5, 7, 12, 19, 31, 50, 81, 131, 212, ...
Since the answer from the previous question is 6, then $N = 6$, and so the N^{th} term in the sequence is the 6^{th} term, which is 50.

P4: There are 100 centimeters in 1 metre, and so 0.9 m = 90 cm.
There are 10 mm in 1 cm, and so N mm $= \frac{N}{10}$ cm.
The total length of rope, in cm, is $43 + 90 + \frac{N}{10} = 133 + \frac{N}{10}$.
Since the answer from the previous question is 50, then $N = 50$, and so
$133 + \frac{N}{10} = 133 + \frac{50}{10} = 133 + 5 = 138$.

ANSWER: $9, 6, 50, 138$

Relay C

P1: The numbers less than 45.63 are 45.61, 45.6, 45.56, 45.1, and 45.53. So there are 5 numbers less than 45.63.

P2: We can remove one square from each side of the balance so we are left with two triangles and one square on the left side, and one circle on the right side.
Now, the mass of one circle is equal to the mass of two triangles and one square, or
$2 \times 4 + N = 8 + N$.
Since the answer from the previous question is 5, then $N = 5$, and so $8 + N = 8 + 5 = 13$.

P3: In N years Magda will be $4 + N$ years old, Kostas will be $6 + N$ years old and Spiro will be $8 + N$ years old. Thus, in N years the sum of their ages will be
$4 + N + 6 + N + 8 + N = 3 \times N + 18$.
Since the answer from the previous question is 13, then $N = 13$, and so
$3 \times 13 + 18 = 39 + 18 = 57$.

P4: Each time Fatemah writes the sentence, she writes the letter "S" 8 times, and the letter "L" 4 times. So she writes the letter "S" $8 - 4 = 4$ more times than the letter "L". If she writes the sentence N times, then she'll write the letter "S" $4 \times N$ more times than the letter "L".
Since the answer to the previous question is 57, then $N = 57$, and so $4 \times N = 4 \times 57 = 228$.

ANSWER: $5, 13, 57, 228$