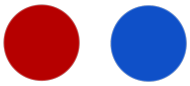

Problem of the Week
Problem E and Solution
Red and Blue Chips

Problem

Jane and Fred each have their own collection of red and blue bingo chips. The ratio of the number of Jane's chips to the number of Fred's chips is $3 : 2$. When they combine their chips, the ratio of the number of red chips to the number of blue chips is $7 : 3$. For Jane's chips, the ratio of the number of red chips to the number of blue chips is $4 : 1$.

What is the ratio of the number of red chips to the number of blue chips for Fred's bingo chips?

Solution

Solution 1

Suppose that Jane and Fred have a total of 100 bingo chips. (We may assume any convenient total number of chips.)

Since the ratio of the number of Jane's chips to the number of Fred's chips is $3 : 2$, then Jane has $\frac{3}{5}$ of the 100 chips, or 60 chips. Fred has the remaining 40 chips.

When they combine their chips, the ratio of the number of red chips to the number of blue chips of $7 : 3$. Therefore, $\frac{7}{10}$ of the 100 chips, or 70 chips, are red and the remaining 30 chips are blue.

For Jane's chips, the ratio of the number of red chips to the number of blue chips is $4 : 1$, so $\frac{4}{5}$ of her 60 chips, or 48 chips, are red and the remaining 12 chips are blue.

Since there are 70 red chips in total, then Fred has $70 - 48 = 22$ red chips.
Since there are 30 blue chips in total, then Fred has $30 - 12 = 18$ blue chips.

Therefore, the ratio of the number of red chips to the number of blue chips for Fred's chips is $22 : 18 = 11 : 9$.

Solution 2

Suppose that Jane and Fred have a total of x chips.

Since the ratio of the number of Jane's chips to the number of Fred's chips is $3 : 2$, then Jane has $\frac{3}{5}$ of the chips, or $\frac{3}{5}x$ chips. Fred has the remaining $\frac{2}{5}x$ chips.

When they combine their chips, the ratio of the number of red chips to the number of blue chips is $7 : 3$. Therefore, $\frac{7}{10}x$ chips are red and the remaining $\frac{3}{10}x$ chips are blue.

For Jane's chips, the ratio of the number of red chips to the number of blue chips is $4 : 1$, so $\frac{4}{5}$ of her $\frac{3}{5}x$ chips, or $\frac{4}{5}\left(\frac{3}{5}x\right) = \frac{12}{25}x$, are red and the remaining $\frac{3}{5}x - \frac{12}{25}x = \frac{3}{25}x$ chips are blue.

Since there are $\frac{7}{10}x$ red chips in total, then Fred has $\frac{7}{10}x - \frac{12}{25}x = \frac{11}{50}x$ red chips.

Since there are $\frac{3}{10}x$ blue chips in total, then Fred has $\frac{3}{10}x - \frac{3}{25}x = \frac{9}{50}x$ blue chips.

Therefore, the ratio of the number of red chips to the number of blue chips for Fred's chips is $\frac{11}{50}x : \frac{9}{50}x = 11 : 9$.