Problem of the Week
Problem E and Solution
Let's Paint

Problem

Painters R Us has been given a large painting job. Initially, Jim started painting by himself. In 15 days, working 9 hours each day, he was able to complete $\frac{3}{8}$ of the job. He decided to have Wanda join him for the remaining part of the job. Together they completed the job in another 10 days, each working 9 hours per day. If Wanda had originally done the job by herself, how many hours would it have taken her to finish the complete job?

Solution

We must make some reasonable assumptions. We will assume that each painter worked at a constant rate each hour, every day. These rates may or may not have been the same for the two painters.

Since Jim completed $\frac{3}{8}$ of the job in 15 days, he would complete $\frac{1}{3}$ of $\frac{3}{8}$, or $\frac{1}{8}$, of the job in 5 days.

Since Jim had completed $\frac{3}{8}$ of the job when Wanda started to work, $\frac{5}{8}$ of the job was left to be completed. Together they completed $\frac{5}{8}$ of the job in 10 days. Since Jim can complete $\frac{1}{8}$ of the job in 5 days, he would have completed $\frac{2}{8}$ of the job in these 10 days. Therefore, Wanda completed the remaining $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$ of the job in these 10 days.

Since Wanda worked 9 hours a day, this means she completed $\frac{3}{8}$ of the job in $10 \times 9 = 90$ hours. Therefore, she completed $\frac{1}{8}$ of the job in 30 hours. Therefore, she could have completed the entire job on her own in $8 \times 30 = 240$ hours.

For Your Information:

Jim completed $\frac{1}{8}$ of the job in 5 days. The whole job could be completed by Jim in $8 \times 5 = 40$ days or 360 hours.

As it was, Jim worked a total of 25 days at 9 hours per day and Wanda worked 10 days at 9 hours per day. They worked a total of $25 \times 9 + 10 \times 9 = 315$ hours.

We know that together Jim and Wanda completed $\frac{5}{8}$ of the job in 10 days. Then, in 2 days they would have completed $\frac{1}{8}$ of the job and in 16 days they would have completed the entire job. That is, working together from the start they would have completed the job in $16 \times 9 = 144$ hours.