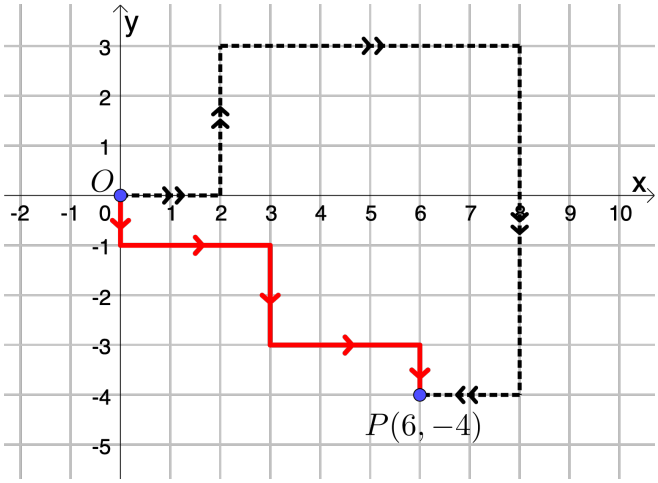
Problem of the Week

Problem E

The Shortest Path

On the Cartesian plane, we draw grid lines at integer points along the x and y axes. We can then draw paths along these grid lines between any two points with integer coordinates. The graph below shows two paths along these grid lines from $O(0, 0)$ to $P(6, -4)$. One path has length 10 and the other has length 20.



There are many different paths along the grid lines from O to P, but the smallest possible length of such a path is 10. Let's call this smallest possible length the *path distance* from O to P.

Determine the number of points with integer coordinates for which the path distance from O to that point is 10.