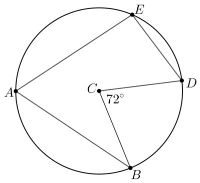


Problem of the Week

Problem E and Solution

Find Another Angle

Problem

The points A, B, D, and E lie on the circumference of a circle with centre C, as shown.

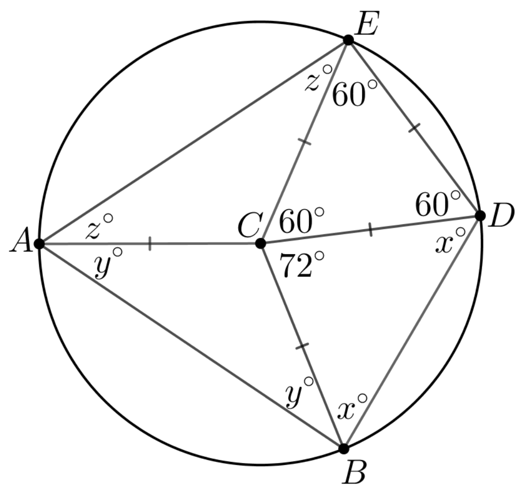
If $\angle BCD = 72°$ and $CD = DE$, then determine the measure of $\angle BAE$.

Solution

We draw radii from C to points A and E on the circumference, and join B to D. Since CA, CB, CD, and CE are all radii, $CA = CB = CD = CE$.

We're given that $CD = DE$. Since $CD = CE$, we have $CD = CE = DE$, and thus $\triangle CDE$ is equilateral. It follows that $\angle ECD = \angle CED = \angle CDE = 60°$.

Let $\angle CDB = x°$, $\angle CBA = y°$, and $\angle CAE = z°$.
Since $CB = CD$, $\triangle CBD$ is isosceles. Therefore, $\angle CBD = \angle CDB = x°$.
Since $CA = CB$, $\triangle CAB$ is isosceles. Therefore, $\angle CAB = \angle CBA = y°$.
Since $CE = CA$, $\triangle CEA$ is isosceles. Therefore, $\angle CEA = \angle CAE = z°$.



Since the angles in a triangle sum to 180°, from $\triangle CBD$ we have $x° + x° + 72° = 180°$. Thus, $2x° = 108°$ and $x = 54$.

Since $ABDE$ is a quadrilateral and the sum of the interior angles of a quadrilateral is equal to 360°, we have

$$\begin{aligned} \angle BAE + \angle ABD + \angle BDE + \angle DEA &= 360° \\ (y° + z°) + (y° + x°) + (x° + 60°) + (60° + z°) &= 360° \\ 2x + 2y + 2z &= 240 \\ x + y + z &= 120 \\ 54 + y + z &= 120 \\ y + z &= 66 \end{aligned}$$

Since $\angle BAE = (y + z)°$, then $\angle BAE = 66°$.